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# FRACTIONS: REDUCING QUADRATICS

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## □ INTRODUCTION

Now that we're getting proficient at factoring, we can knock some ugly fractions down to size. Let's review a few reducing problems to get us ready for this chapter.

First Fraction: Let's reduce the fraction  $\frac{39}{52}$ . By prime factoring the top and bottom, we get  $\frac{39}{52} = \frac{3 \cdot 13}{2 \cdot 2 \cdot 13} = \frac{3 \cdot \cancel{13}^1}{2 \cdot 2 \cdot \cancel{13}_1} = \frac{3}{4}$ . Once the top and bottom have been factored (i.e., written as a product), we can divide out (or cancel) the common factor of 13.

Second Fraction: Applying the same logic to the fraction  $\frac{ax}{a^2 + 7a}$ , we can reduce it by writing  $\frac{ax}{a^2 + 7a} = \frac{ax}{a(a+7)} = \frac{\cancel{a}x}{\cancel{a}(a+7)} = \frac{x}{a+7}$ .

And a Third Fraction:  $\frac{LT - QT}{AL - AQ} = \frac{\overbrace{L(L-Q)}^{\text{top factored}}}{\underbrace{A(L-Q)}_{\text{bottom factored}}} = \frac{\cancel{L(L-Q)}^1}{\cancel{A(L-Q)}_1} = \frac{T}{A}$

And don't forget that when something is factored, it consists of a single term (where multiplication is the final operation). Thus,  $a(b - c)$  is factored, but  $nx + ny$  is not factored.

## ❑ REDUCING QUADRATIC FRACTIONS

Whether we're reducing  $\frac{39}{52}$  (like the one above) or something like what you're about to see, the process is the same:

1. Factor top and bottom.
2. Divide out any common factors.

**EXAMPLES:** Reduce each fraction to lowest terms:

$$A. \quad \frac{x^2 + 5x + 6}{x^2 - 4} = \frac{(x+3)(x+2)}{(x+2)(x-2)} = \frac{\cancel{(x+2)}(x+3)}{\cancel{(x+2)}(x-2)} = \frac{x+3}{x-2}$$

$$B. \quad \frac{6n^2 + 5n - 21}{3n^2 + 22n + 35} = \frac{(3n+7)(2n-3)}{(3n+7)(n+5)} = \frac{\cancel{(3n+7)}(2n-3)}{\cancel{(3n+7)}(n+5)} = \frac{2n-3}{n+5}$$


$$C. \quad \frac{x+5}{2x^2 + 7x - 15} = \frac{x+5}{(2x-3)(x+5)} = \frac{\cancel{x+5}}{(2x-3)\cancel{(x+5)}} = \frac{1}{2x-3}$$

$$D. \quad \frac{3w^2 - 2w - 5}{3w - 5} = \frac{(3w-5)(w+1)}{3w-5} = \frac{\cancel{(3w-5)}(w+1)}{\cancel{3w-5}} = w+1$$

$$E. \quad \frac{a^2 - 25}{a^2 + 5a + 6} = \frac{(a+5)(a-5)}{(a+3)(a+2)} \text{ which contains no common factors.}$$

Therefore, this fraction is **not reducible**.

EVERY TIME YOU DO THIS:



$$f(x) = \frac{\cancel{x^2} + 2x + 1}{\cancel{x^2} + 3}$$

$$= \frac{2x+1}{3}$$

A KITTEN DIES.

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## Homework

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1. Verify that the fraction  $\frac{ab+ac}{ad}$  reduces to  $\frac{b+c}{d}$  by letting  $a = 2$ ,  $b = 3$ ,  $c = 4$ , and  $d = 6$  in both fractions.
  
2. Prove that the fraction  $\frac{x+z}{yz}$  does not reduce to  $\frac{x}{y}$  by choosing numerical values for  $x$ ,  $y$ , and  $z$ . Also prove to yourself that  $\frac{x+z}{yz}$  does not reduce to  $\frac{x+1}{y}$ , either.
  
3. Reduce each fraction to lowest terms:
 

a. $\frac{n^2 - 9}{n^2 + 6n + 9}$	b. $\frac{4a^2 + 4a + 1}{2a^2 + 5a + 2}$	c. $\frac{c + 7}{c^2 - 49}$
d. $\frac{4w^2 - 9}{2w - 3}$	e. $\frac{6x^2 + 11x - 7}{2x^2 + 17x - 9}$	f. $\frac{h^2 + 3h + 2}{h^2 - 3h + 2}$
g. $\frac{3k^2 - 17k + 1}{3k^2 - 17k + 1}$	h. $\frac{100 - w^2}{w^2 + 10w}$	i. $\frac{10x^2 + 11x - 6}{5x^2 - 12x + 4}$
j. $\frac{x^2 + 7x + 10}{x^2 + 3x + 2}$	k. $\frac{n^2 - 9}{n^2 + 4n + 3}$	l. $\frac{y - 7}{y^2 - 14y + 49}$
m. $\frac{w^2 - 81}{w + 9}$	n. $\frac{m^2 + 10m + 25}{m^2 - 25}$	o. $\frac{x^2 + 2x + 1}{x^2 - 4}$
p. $\frac{6a^2 + 13a - 5}{9a^2 + 12a - 5}$	q. $\frac{k^2 - 6k + 7}{7 - 6k + k^2}$	r. $\frac{16u^2 + 34u - 15}{2u^2 + 3u - 5}$

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## Review Problems

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|---|--------------------------------------|
| 4. Reduce: $\frac{6a^2 - 5a - 21}{3a^2 - 4a - 7}$     | 5. Reduce: $\frac{x^2 - 9}{x^2 - 4}$ |
| 6. Reduce: $\frac{10a^2 + 29a - 21}{5a^2 - 38a + 21}$ | 7. Reduce: $\frac{x - 3}{x^2 - 9}$   |
| 8. Reduce: $\frac{n^2 + 14n + 49}{(n + 7)^2}$         |                                      |

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## Solutions

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1. Note that this does not constitute a complete proof.
  2. Be sure you don't let either the  $y$  or the  $z$  be zero. How come?
- 3.
- |                       |                       |                      |                        |
|-----------------------|-----------------------|----------------------|------------------------|
| a. $\frac{n-3}{n+3}$  | b. $\frac{2a+1}{a+2}$ | c. $\frac{1}{c-7}$   | d. $2w + 3$            |
| e. $\frac{3x+7}{x+9}$ | f. Not reducible      | g. 1                 | h. $\frac{10-w}{w}$    |
| i. $\frac{2x+3}{x-2}$ | j. $\frac{x+5}{x+1}$  | k. $\frac{n-3}{n+1}$ | l. $\frac{1}{y-7}$     |
| m. $w - 9$            | n. $\frac{m+5}{m-5}$  | o. Not Reducible     | p. $\frac{2a+5}{3a+5}$ |
| q. 1                  | r. $\frac{8u-3}{u-1}$ |                      |                        |

4.  $\frac{2a+3}{a+1}$       5. Not reducible      6.  $\frac{2a+7}{a-7}$
7.  $\frac{1}{x+3}$       8. 1

### □ **TO $\infty$ AND BEYOND**

- A. Consider the fraction

$$\frac{3}{x-7}$$

Let's choose a number for  $x$ . We see that if  $x = 4$ , for instance, then the value of the fraction is

$$\frac{3}{4-7} = \frac{3}{-3} = -1$$

And  $x$  can also take on the value 0, since  $\frac{3}{0-7} = \frac{3}{-7} = -\frac{3}{7}$ . In fact,  $x$  can be any number at all . . . with one exception. What is that number?

- B. Consider the fraction

$$\frac{x^2 - 4}{x^2 - 11x + 30}$$

Find the values of  $x$  which must never be used in this fraction.

*“Education is what survives when what has been learned has been forgotten.”*

– B. F. Skinner (1904 - 1990)

