USING THE GCF TO REDUCE FRACTIONS

❑ *REDUCING ALGEBRAIC FRACTIONS*

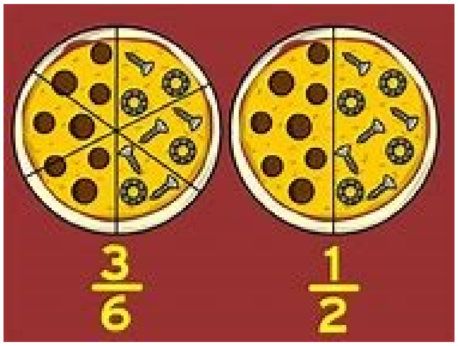
Most of us learned to reduce an arithmetic fraction by dividing the top and the bottom of the fraction by the same (non-zero) number. For example,

$$\frac{30}{75} = \frac{30 \div \mathbf{15}}{75 \div \mathbf{15}} = \boxed{\frac{2}{5}}$$

When it comes to algebra, though, we need to look at reducing a fraction a little differently, since it's kind of hard to divide letters by letters. So now we re-examine the reducing of arithmetic fractions so that we get a method more appropriate to algebraic fractions. Watch as we reduce the same fraction by *factoring*:

$$\frac{30}{75} = \frac{2\cdot 3\cdot 5}{3\cdot 5\cdot 5} = \frac{2\cdot \cancel{3}\cdot \cancel{5}}{\cancel{3}\cdot \cancel{5}\cdot 5} = \boxed{\frac{2}{5}}$$

which, of course, is the same answer as before. Here's what we did: First we factored the top and the bottom into prime factors -- the very building blocks of the whole numbers. Then we "divided out" any factor that was common to both the top and the bottom (since any number divided by itself is 1). Whatever factors remain constitute the final answer.



EXAMPLE 1: **Reduce each fraction:**

A. $\dfrac{18}{54} = \dfrac{2\cdot3\cdot3}{2\cdot3\cdot3\cdot3} = \dfrac{\cancel{2}\cdot\cancel{3}\cdot\cancel{3}}{\cancel{2}\cdot\cancel{3}\cdot\cancel{3}\cdot3} = \mathbf{\dfrac{1}{3}}$

Notice that we don't leave a zero in the numerator just because every factor in the numerator canceled out. Remember that canceling is really dividing, so each time we cancel a pair of factors, we're really dividing a number by itself, which is always 1.

B. $\dfrac{ab}{a} = \dfrac{\cancel{a}b}{\cancel{a}} = \boldsymbol{b}$

Because a and b are being multiplied (they're one term), we can divide top and bottom by a.

C. $\dfrac{bc}{cw} = \dfrac{b\cancel{c}}{\cancel{c}w} = \boldsymbol{\dfrac{b}{w}}$

The top and bottom each consist of one term, so we can divide top and bottom by c.

EXAMPLE 2: **Reduce:** $\boldsymbol{\dfrac{a+b}{a+c}}$

Solution: Can we divide top and bottom by a? I don't think so. But maybe you think so; give me a moment to explain. Since dividing is designed to remove multiplication, and since the top and bottom have addition in them (they're each two terms), we cannot cancel the a's. Therefore, this fraction is **not reducible**.

Here's a more concrete way to verify this fact. If you think you should be able to cancel the a's in this fraction, let's do it and see what happens when we use numbers. Suppose $a = 6$, $b = 10$, and $c = 12$. Using the Order of Operations, we would get

$$\frac{a+b}{a+c} = \frac{6+10}{6+12} = \frac{16}{18} = \frac{8}{9}$$

On the other hand, canceling the a's completely would produce the following outcome:

$$\frac{a+b}{a+c} = \frac{b}{c} = \frac{10}{12} = \frac{5}{6}$$

And if you think that it would be legal to divide out the a's and leave 1's in their place, the following would be the consequence:

$$\frac{a+b}{a+c} = \frac{1+b}{1+c} = \frac{1+10}{1+12} = \frac{11}{13}$$

That's three different answers, depending upon our theory of reducing fractions. Which one's right? Well, since the Order of Operations has been with us for a long time now, I prefer to stick with what I know, and conclude that the only valid answer is $\frac{8}{9}$. I therefore think we should reject all this goofy canceling; none of it worked.

The only time we can "cancel" in a fraction is when both the top and the bottom consist of <u>one</u> term (that is, the final operation is multiplication on the top and on the bottom).

Since reducing a fraction requires one term on the top and one term on the bottom, we can now utilize our knowledge of factoring to reduce fractions which may have more than one term on the top or bottom.

<u>EXAMPLE 3:</u> **Reduce each fraction:**

A. $\frac{2x+8}{2a-10} = \frac{2(x+4)}{2(a-5)} = \frac{\cancel{2}(x+4)}{\cancel{2}(a-5)} = \boldsymbol{\frac{x+4}{a-5}}$

B. $\frac{ax+bx}{xy-xz} = \frac{x(a+b)}{x(y-z)} = \frac{\cancel{x}(a+b)}{\cancel{x}(y-z)} = \boldsymbol{\frac{a+b}{y-z}}$

C. $\frac{bn+an}{aq+bq} = \frac{n(b+a)}{q(a+b)} = \frac{n(a+b)}{q(a+b)} = \frac{n\cancel{(a+b)}}{q\cancel{(a+b)}} = \boldsymbol{\frac{n}{q}}$

D. $\dfrac{ux - uw}{ax + aw} = \dfrac{u(x-w)}{a(x+w)}$

There's no common factor to cancel. So the original fraction is **not reducible**.

EXAMPLE 4: **Reduce each fraction:**

A. $\dfrac{x^2 + x}{x} = \dfrac{x(x+1)}{x} = \dfrac{\cancel{x}(x+1)}{\cancel{x}} = \boldsymbol{x+1}$

B. $\dfrac{n^2 - n}{n-1} = \dfrac{n(n-1)}{n-1} = \dfrac{n\cancel{(n-1)}}{\cancel{n-1}} = \boldsymbol{n}$

C. $\dfrac{a}{a^2 - 3a} = \dfrac{a}{a(a-3)} = \dfrac{\cancel{a}}{\cancel{a}(a-3)} = \dfrac{\mathbf{1}}{\boldsymbol{a-3}}$

D. $\dfrac{u-4}{u^2 - 4u} = \dfrac{1(u-4)}{u(u-4)} = \dfrac{1\cancel{(u-4)}}{u\cancel{(u-4)}} = \dfrac{\mathbf{1}}{\boldsymbol{u}}$

We now have a technique for reducing fractions to lowest terms, but it may still be a little hard to believe, for instance, that $\dfrac{n^2 - n}{n-1} = n$, as in part B of the previous example. Perhaps you'll feel a little more confident in this answer if we substitute a number for n and verify the equality ourselves. So let's choose $n = 10$; then

$$\frac{n^2 - n}{n-1} = \frac{\mathbf{10}^2 - \mathbf{10}}{\mathbf{10} - 1} = \frac{100 - 10}{10 - 1} = \frac{90}{9} = 10, \text{ which equals } n. \; ☺$$

The Steps Needed to Reduce a Fraction:

1. Be sure the top is factored.
2. Be sure the bottom is factored.
3. Divide top and bottom by any common factors.

Homework

1. Reduce each fraction by factoring into primes:

a. $\frac{20}{22}$ b. $\frac{17}{51}$ c. $\frac{84}{174}$ d. $\frac{22}{33}$ e. $\frac{20}{41}$

f. $\frac{30}{50}$ g. $\frac{34}{51}$ h. $\frac{13}{39}$ i. $\frac{26}{65}$ j. $\frac{32}{128}$

2. Reduce each fraction:

a. $\frac{3x-12}{3n+21}$ b. $\frac{ax+bx}{ay+by}$ c. $\frac{tx+tz}{ty-tz}$

d. $\frac{xy+yz}{ay-by}$ e. $\frac{aR+aT}{bR+bT}$ f. $\frac{mx+my}{ax-ay}$

g. $\frac{ax+bx}{ax-cx}$ h. $\frac{ax-bx}{ay-by}$ i. $\frac{a}{am+an}$

j. $\frac{PT-QT}{T}$ k. $\frac{gn+hn}{gn-hn}$ l. $\frac{ab-ac}{cx-xy}$

3. Reduce each fraction:

a. $\frac{x^2+3x}{x}$ b. $\frac{n^2-n}{n-1}$ c. $\frac{z}{z+z^2}$ d. $\frac{Q-3}{QR-3R}$

e. $\frac{y^2-y}{y}$ f. $\frac{a^2+a}{a+1}$ g. $\frac{t-1}{t^2-t}$ h. $\frac{n}{n^2+9n}$

i. $\frac{a-3}{a^2-3a}$ j. $\frac{c+2}{c^2+c}$ k. $\frac{z^2-3z}{z-3}$ l. $\frac{Q^2-10Q}{Q}$

m. $\frac{c}{ac-c^2}$ n. $\frac{rx+x^2}{r+1}$ o. $\frac{ax+bx^2}{xy-xz^2}$ p. $\frac{an+b}{n}$

Review Problems

4. Reduce each fraction:

a. $\frac{5x+20}{10x-45}$ b. $\frac{ax-bx}{cx+dx}$ c. $\frac{am+an}{a}$ d. $\frac{w}{w^2-3w}$

e. $\frac{t^2+t}{t+1}$ f. $\frac{R^2-4R}{R}$ g. $\frac{2x+8}{3x-12}$ h. $\frac{ax+ay}{bx+by}$

5. Reduce each fraction:

a. $\frac{5x+10}{10x+45}$ b. $\frac{a^2-ab}{ac+ad}$ c. $\frac{cm+cn}{c}$ d. $\frac{u}{u^2+7u}$

e. $\frac{t-1}{t^2-t}$ f. $\frac{R^2+99R}{R}$ g. $\frac{2x+8}{6x-12}$ h. $\frac{ax+ay}{bx-by}$

Solutions

1. a. $\frac{20}{22} = \frac{\not{2}\cdot 2\cdot 5}{\not{2}\cdot 11} = \frac{10}{11}$ b. $\frac{1}{3}$ c. $\frac{14}{29}$ d. $\frac{2}{3}$ e. $\frac{20}{41}$

f. $\frac{3}{5}$ g. $\frac{2}{3}$ h. $\frac{1}{3}$ i. $\frac{2}{5}$ j. $\frac{1}{4}$

2. a. $\frac{x-4}{n+7}$ b. $\frac{x}{y}$ c. $\frac{x+z}{y-z}$ d. $\frac{x+z}{a-b}$

e. $\frac{a}{b}$ f. Not reducible g. $\frac{a+b}{a-c}$ h. $\frac{x}{y}$

i. $\frac{1}{m+n}$ j. $P-Q$ k. $\frac{g+h}{g-h}$ l. Not reducible

3. a. $x + 3$ b. n c. $\frac{1}{1+z}$ d. $\frac{1}{R}$

e. $y - 1$ f. a g. $\frac{1}{t}$ h. $\frac{1}{n+9}$

i. $\frac{1}{a}$ j. Not reducible k. z l. $Q - 10$

m. $\frac{1}{a-c}$ n. Not reducible o. $\frac{a+bx}{y-z^2}$ p. Not reducible

4. a. $\frac{x+4}{2x-9}$ b. $\frac{a-b}{c+d}$ c. $m + n$ d. $\frac{1}{w-3}$

e. t f. $R - 4$ g. Not reducible h. $\frac{a}{b}$

5. a. $\frac{x+2}{2x+9}$ b. $\frac{a-b}{c+d}$ c. $m + n$ d. $\frac{1}{u+7}$

e. $\frac{1}{t}$ f. $R + 99$ g. $\frac{x+4}{3x-6}$ h. Not reducible

"I'd plead the fifth, but we haven't learned fractions yet."

“Education is not received.

It is achieved.”

– Anonymous