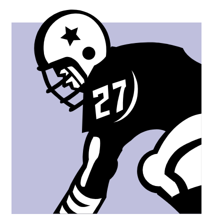
FUNCTIONS: TABLES AND MAPPINGS

❑ INTRODUCTION

The notion of a *function* is much more abstract than most of the algebra concepts you've seen so far, so we'll start with three specific non-math examples.

❑ THE MEANING OF A FUNCTION VIA TABLES



First Example: A football game is divided into four quarters. The following table shows each quarter together with the total number of points scored during that quarter.

Quarter	Number of Points Scored
1st	21
2nd	9
3rd	0
4th	25
Inputs	Outputs

This table is a ***function***, and here's why:

First, we have a set of **inputs**, the four quarters:

1st 2nd 3rd 4th

Second, we have a set of **outputs**, the total points scored in each quarter:

21 9 0 25

Third, there's a definite connection, or correspondence, between the inputs and the outputs. For example, the input "2nd" is associated with the output "9." What about the input "4th"? The output is "25." Similarly, "21" is the output for the input "1st," while "3rd" produces the output "0."

Fourth, and most importantly, notice that each input has exactly one output. For instance, the input "2nd" has exactly one output, namely "9." There can be no argument that the output is 9 -- just look at the table. Also, the input "4th" is clearly has the output "25," and nothing else. This is the essence of a function, and so our table of football quarters and points scored is a function.

Second Example: So, what isn't a function? Consider the following table:

City	Major League Baseball Teams
San Francisco	Giants
Oakland	Athletics
New York	Yankees
New York	Mets
Inputs	Outputs

The input San Francisco has exactly one output, Giants, because the city of San Francisco is home to exactly one major league baseball team. The input Oakland also has exactly one output associated with it. So far so good – in terms of being a function.

But check out New York: It has two baseball teams, the Yankees and the Mets. We thus have an input (New York) with more than one output (Yankees and Mets). This kills the function concept, and we conclude that our table of cities and baseball teams is not a function.

To summarize this example, for something to be a function, there must be exactly one output for each input; this example has an input with two outputs. That's why the city/team pairings do NOT make a function.

Third Example:

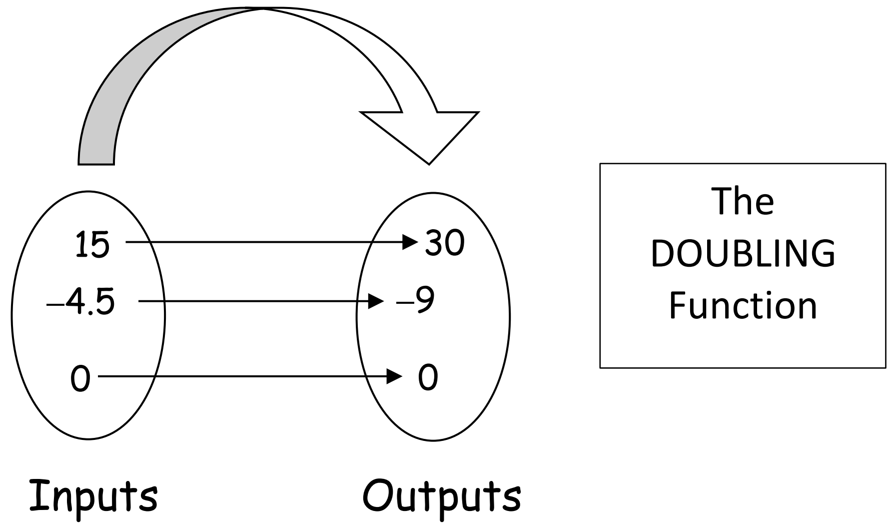
City	Population
Elmville	35,000
Gomerville	47,500
Moeville	35,000
Inputs	Outputs

Is this table a function? Does each input have exactly one output? YES, and I'll prove it to you. The input Elmville has one output (35,000), the input Gomerville has one output (47,500), and the input Moeville has one output (35,000). Does it matter that Elmville and Moeville have the same population? Not at all -- it's just a coincidence. The fact is, every city has exactly one population associated with it; that is, each input has exactly one output. We must conclude that the population table is a function.

❑ *MATHEMATICAL EXAMPLES OF FUNCTIONS VIA MAPPINGS*

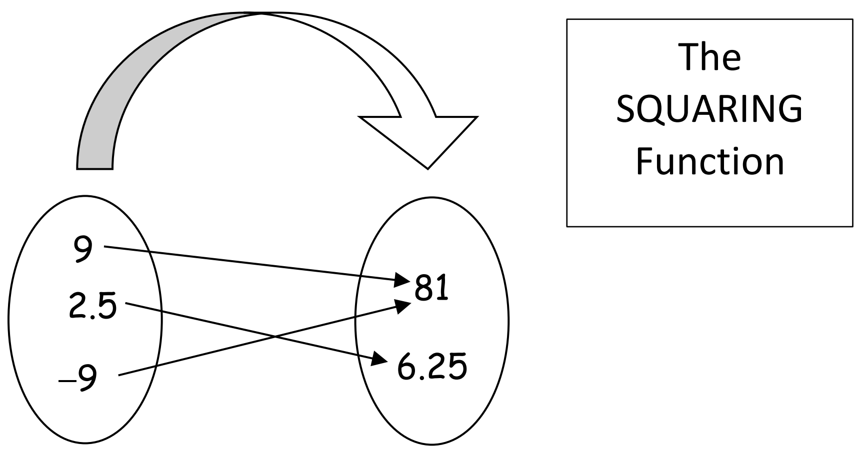
EXAMPLE 1: **Analyze the *doubling* function.**

Solution: Consider the ***doubling function***. This function takes an input, and produces an output that is double the input. For example, if the input is 15, then the output is double 15, or 30. That is, this function takes an input of 15 and produces an output of 30. In the diagram below we see three inputs (15, –4.5, and 0) producing three outputs (30, –9, and 0).



EXAMPLE 2:

A. Look at the ***squaring*** function.



Why is this a function? Because, given an input, there is exactly one output for it. Notice that even though both 9 and –9 produce the same output of 81, it nevertheless is still the case that 9 squared is 81, and only 81 -- and that –9 squared is 81, and only 81. Each input has exactly one output: *squaring* is a function.

B. Consider the ***square root*** function, $\sqrt{x}$. An input of 49 produces an output of 7, and an input of 2 produces an output of $\sqrt{2}$. Recall that the formula represents the non-negative square root only.

What inputs are allowed in this function? Since negative numbers don't have real-number square roots, all inputs must be at least zero.

To reiterate, this is a function because given any legal input ($x \geq 0$), we obtain exactly <u>one</u> output.

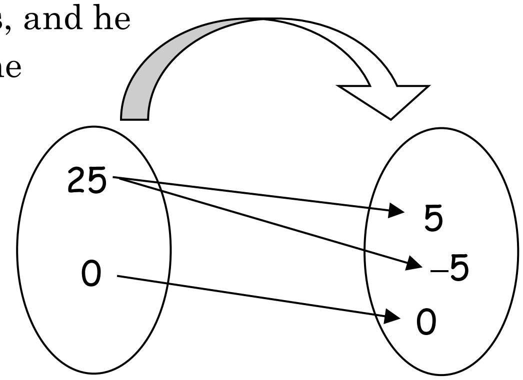
C. Now for the ***reciprocal*** function; that is, $\frac{1}{x}$. Thus,

An input of 7 produces $\frac{1}{7}$, an input of $-\frac{2}{3}$ produces $-\frac{3}{2}$, and an input of 0 produces nothing at all, since 0 has no reciprocal. That is, 0 is not a legal input to the reciprocal function.

What can x be in this function? Well, the only number we can't divide by is 0, so x can be any real number except 0.

Starting with any legal input ($x \neq 0$), we obtain exactly one output -- we conclude that the reciprocal is a function.

E. Now consider the operation "**take <u>both</u> square roots**": $\pm\sqrt{x}$. Is this a function? Suppose we take the input $x = 25$. Ask a person what the output is, and he might say 5, since 5 is indeed the positive square root of 25. But ask another person, and she might say –5, since –5 is certainly the negative square root of 25. Who's right? They're both right! There's nothing illegal or immoral about our formula. The fact is,



it's simply <u>not</u> a function -- there's an input that produces more than one output.

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Homework

1. Construct a table of inputs and outputs which represents a function.

2. Construct a table of inputs and outputs which does not represent a function.

3. Construct a mapping (the circles and arrows) which demonstrates "absolute value." Is it a function? Explain.

4. Construct a mapping which represents a <u>non</u>-function. Explain precisely why it's not a function.

5. Using the functions in Example 2, calculate the output for the three given inputs:

   a. The *squaring* function: 2/3 –15 π
   Is there any number which cannot be used as an input?

   b. The (non-negative) *square root* function: 144 1 5
   Is there any number which cannot be used as an input?

   c. The *reciprocal* function: 1/4 –3 1
   Is there any number which cannot be used as an input?

6. True/False:

   a. In the tripling function, if the input is 12, the output is 36.

   b. In the cubing function, if the output is 125, the input is 5.

Solutions

1. Be sure that every input has exactly one output.

2. Be sure that at least one input has more than one output.

3. I'd like to see what you came up with.

4. Be sure you have at least one input with 2 or more arrows diverging from it.

5.

	a.	4/9	225	π^2	Every number can be used as an input.
	b.	12	1	$\sqrt{5}$	No negative number can be square-rooted.
	c.	4	–1/3	1	The only number without a reciprocal is 0.

6. a. T b. T

"The secret of getting ahead is getting started. The secret of getting started is breaking your complex overwhelming tasks into small manageable tasks, and then starting on the first one."

Mark Twain