INTRO TO GEOMETRY

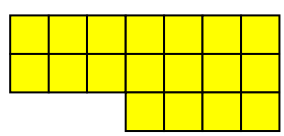
❑ INTRODUCTION

*G**eo*: Greek for earth, and *metros*: Greek for measure. These roots are the origin of the word "geometry," which literally means "earth measurement." The study of geometry has gone way beyond the notions of triangles and circles, however -- from the shapes of molecules to the structure of 4-dimensional space-time.

Area

The ***area*** of a geometric figure is a measure of how big a region is enclosed inside the figure.

For example, the area of the region below is 18 square units. Just count the number of little squares, and you've got the **area**.

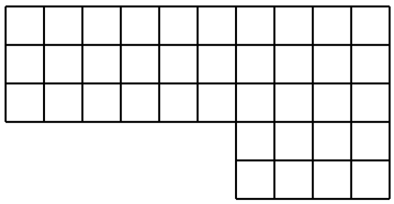


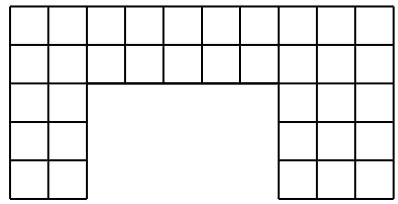
Perimeter

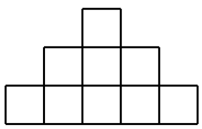
The distance around a geometric figure is called its ***perimeter***. For example, the perimeter of the region above is 20 units. Start anywhere you like, and march along the edge of the region until you reach your starting point. The distance you've traveled is the **perimeter**. To help you remember this, note that *peri* means "around," as in *peri*scope or *peri*pheral vision.

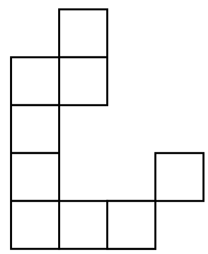
Homework

Find the area and perimeter of each geometric shape:

1. 

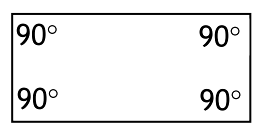
2. 

3. 

4. 

❑ *RECTANGLES AND SQUARES*

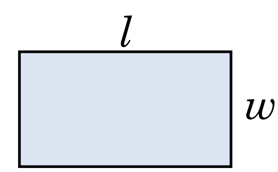
In Manhattan, Fifth Avenue (which runs north-south) meets 42nd Street (which runs east-west) at a 90° angle. The floor and the wall also meet at a 90° angle.



A ***rectangle*** is a four-sided figure with all inside angles equal to 90°. This implies that adjacent sides (sides next to each other) are *perpendicular* and opposite sides are *parallel*. Notice that a ***square*** (where all four sides have the same length) is also a four-sided figure with all four inside angles equal to 90°. Therefore, by definition, <u>a square is a special kind of rectangle</u>. We can conclude that every square is a rectangle, but certainly not every rectangle is a square.

Note also that because of all the 90° angles, it follows that the top and bottom of the rectangle are the same length, and the left and right sides are the same length. In other words, each pair of opposite sides of a rectangle have the same length.

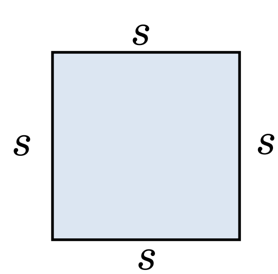
We know that the distance around the rectangle (the sum of all four of its sides) is called its ***perimeter***. If l is the length of the rectangle and w is the width, then the perimeter is $P = l + l + w + w = 2l + 2w$. The ***area*** of a rectangle is a measure of the size of the region enclosed by the rectangle. The formula for the area is $A = lw$.



Perimeter: $P = 2l + 2w$

Area: $A = lw$

As for the square, we can see that the perimeter, the distance around, is simply four s's added together: $s + s + s + s$, or $\mathbf{4s}$. The area of a square, since it's a special rectangle, is just the length times the width; but the length and the width are both s, so the area is $s \times s$, or $\mathbf{s^2}$.



Perimeter: $P = 4s$

Area: $A = s^2$

Homework

5. Find the perimeter of each rectangle given the length and width:

a. $l = 23 \quad w = 42$ b. $l = 34.7 \quad w = 1.22$

c. $l = 7 \quad w = \frac{4}{5}$ d. $l = 2\frac{3}{4} \quad w = 4\frac{1}{2}$

6. Find the area of each rectangle given the length and width:

a. $l = 23 \quad w = 42$ b. $l = 34.7 \quad w = 1.22$

c. $l = 7 \quad w = \frac{4}{5}$ d. $l = 2\frac{3}{4} \quad w = 4\frac{1}{2}$

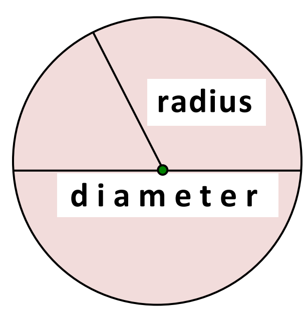
7. Each side of a square is 22. Find the square's perimeter and its area.

8. If each side of a square is 23.7, find its perimeter and area.

9. If each side of a square is $\frac{4}{5}$, find its perimeter and area.

❑ *CIRCLES*

Basic Definitions



The distance from any point on a circle to the center of the circle is called its ***radius***. The distance from one point on a circle to another point on the circle, through the center of the circle, is called its ***diameter***. It should be clear that the diameter is twice the radius:

Diameter: $d = 2r$

Equivalently, the radius is half the diameter $r = \frac{1}{2}d$, which can also be written as

Radius: $$r = \frac{d}{2}$$

The ***circumference*** of a circle is the distance all the way around the circle -- it's the circle's *perimeter*. The ***area*** of a circle is a measure of the region inside the circle. The formulas for circumference and area will be stated after a discussion of a very important number.

A Very Special Number

Choose any circle at all -- tiny, mid-sized, or gigantic. When you divide its circumference by its diameter -- no matter the circle -- you always end up with the same number, a constant a little bigger than 3. This quotient, (the circumference divided by the diameter), which is the same for all circles, is denoted by the Greek letter pi, "**π**", from the Greek word "perimetros" -- though the Greeks themselves did not use the symbol π for this special number. In other words,

The modern symbol for pi, π was first used in 1706 by William Jones.

$$\pi = \frac{C}{d} \quad \text{(definition of the number } \pi\text{)}$$

We also say that π is the ***ratio*** of the circumference to the diameter of any circle. To reiterate:

Given any circle, of any size, anywhere in the whole universe, the ratio of its circumference to its diameter is always the same number: π.

The decimal version of π has an infinite number of digits in it (very difficult to prove), and therefore any decimal number we write for π will simply be an approximation. The calculator gives something like 3.141592654 -- two useful approximations of π

The circumference of any circle is a little over 3 times its diameter.

are 3.14 and $\frac{22}{7}$. But since this is an algebra class, we'll usually stick with the exact value of π, which is, of course, simply written π.

Formulas for Circumference and Area

The circumference of a circle can be found by calculating 2 times π times the radius (this formula will be derived later in a later chapter):

Circumference: $\boldsymbol{C = 2\pi r}$

As for area, the formula can be derived in a later course, so we simply state it and use it:

Area: $\boldsymbol{A = \pi r^2}$

Note #1: Since the Order of Operations specifies that exponents have priority over multiplication, we note that πr^2 tells us to first square the r, and then multiply by π.

Note #2: Students sometimes mix up the formulas for circumference and area. Here's a way to remember which is which. The area of a geometric shape is always in square units, for example, square feet -- and which quantity, $2\pi r$ or πr^2, has the "square" in it? The πr^2 does, of course. So area is πr^2, and circumference is the one without the square in it, $2\pi r$.

Note #3: No teacher intends to confuse you, but it happens. Your Pre-Algebra teacher may have taught you that the circumference of a circle is given by the formula $C = \pi d$, instead of the formula $C = 2\pi r$ stated above. They are really the same formula, since $d = 2r$, so you can certainly use whichever one you like, though in this course we'll always use $C = 2\pi r$.

EXAMPLE 1:

A. **The radius of a circle is 89. Find the diameter.**

Since $d = 2r$, it follows that $d = 2(89) = \mathbf{178}$

B. **The diameter of a circle is 13. Find the radius.**

The formula $r = \frac{d}{2}$ tells us that $r = \frac{13}{2} = \mathbf{6\frac{1}{2}}$, or **6.5**

EXAMPLE 2: **The radius of a circle is 17.5. Find the circumference.**

Solution: Using the formula for the circumference of a circle, we proceed as follows:

$$C = 2\pi r \quad \text{(the circumference formula)}$$

$$\Rightarrow \quad C = 2\pi\,(\mathbf{17.5}) \quad \text{(plug in the given radius for } r\text{)}$$

$$\Rightarrow \quad C = 2(17.5)\,\pi \quad \text{(rearrange the factors)}$$

$$\Rightarrow \quad C = \boxed{35\pi} \quad \text{(multiply 2 by 17.5)}$$

Note that this is the exact answer, because the symbol π is in the final answer. An approximation could easily be found by using 3.14, for instance, for π, and then multiplying 35 by 3.14 to get **109.9**.

EXAMPLE 3: **The radius of a circle is 17. Find the area.**

Solution: The relevant formula is $A = \pi r^2$. Notice that only the r is being squared in this formula, since the Order of Operations specifies that exponents have priority over multiplication. We therefore calculate the area as follows:

$$A = \pi r^2 \quad \text{(the area of a circle formula)}$$

$$\Rightarrow \quad A = \pi\,(\mathbf{17})^2 \quad \text{(plug in the given radius for } r\text{)}$$

$$\Rightarrow \quad A = \pi\,(289) \quad \text{(square the 17, and we're basically done)}$$

$$\Rightarrow \quad A = \boxed{289\pi} \quad \text{(it looks prettier this way)}$$

Homework

The Circle Formulas

$$\pi = \frac{C}{d} \qquad d = 2r \qquad r = \frac{d}{2} \qquad C = 2\pi r \qquad A = \pi r^2$$

10. a. The radius of a circle is 0.023. What is the ratio of its circumference to its diameter?

    b. The diameter of a circle is $\frac{123}{967}$. What is the ratio of its circumference to its diameter?

    c. The circumference of a circle is 2134.909. What is the ratio of its circumference to its diameter?

    d. The area of a circle is $11.178\pi^3$. What is the ratio of its circumference to its diameter?

11. For each problem, find the diameter if the radius is given, and find the radius if the diameter is given:

a. $r = 20$ b. $r = 7.6$ c. $r = \frac{1}{7}$ d. $r = 0.5$

e. $d = 44$ f. $d = 0.25$ g. $d = 17$ h. $d = \frac{1}{5}$

12. Find the **circumference** of the circle with the given radius; leave your answers in exact form (that is, leave π in the answer):

a. $r = 8$ b. $r = 33.4$ c. $r = 0.07$ d. $r = 89$

e. $r = 13$ f. $r = 0.5$ g. $r = 100$ h. $r = 77.5$

13. Find the **area** of the circle with the given radius -- leave your answers in exact form (that is, leave **π** in the answer):

a. $r = 9$ b. $r = 100$ c. $r = 0.3$ d. $r = 3.5$

e. $r = 10$ f. $r = 2.5$ g. $r = 1$ h. $r = 0.08$

❑ *A Biblical View of π*

From I Kings 7:23 comes the sentence:

> "*Then He made the molten sea, ten cubits from brim to brim, while a line of 30 cubits measured it around.*"

The word "sea" refers to a large container for holding water. "Brim to brim" refers to the diameter of 10 cubits (the cubit was an ancient measurement of about 20 inches), while the 30 cubits refers to the circumference. Thus, from this quote we derive the value $\pi = \frac{C}{d} = \frac{30 \text{ cubits}}{10 \text{ cubits}} = 3$, which is quite a good estimate, given how old the description is.

Review Problems

14. The length of a rectangle is 55 and its width is 44. Find the perimeter of the rectangle.
15. The length of a rectangle is 34 and its width is 22. Find its area.
16. Find the perimeter and area of a square each of whose sides is 25.
17. Given any circle of any size, what is the ratio of its circumference to its diameter? That is, what is $\frac{C}{d}$ for any circle?
18. The radius of a circle is 15. What is its diameter?
19. The diameter of a circle is 88. Find the radius.
20. The radius of a circle is 7. Find the area in exact form.
21. The radius of a circle is 12. Find the circumference in exact form.

Solutions

1. $A = 38$; $P = 30$

2. $A = 35$; $P = 36$

3. $A = 9$; $P = 16$

4. $A = 9$; $P = 22$

5. a. 130 b. 71.84 c. $15\frac{3}{5}$ d. $14\frac{1}{2}$

6. a. 966 b. 42.334 c. $5\frac{3}{5}$ d. $12\frac{3}{8}$

7. $P = 88$ $A = 484$

8. $P = 94.8$ $A = 561.69$

9. $P = 3\frac{1}{5}$ $A = \frac{16}{25}$

10. a. π b. π c. π d. π

11. a. $d = 40$ b. $d = 15.2$ c. $d = \frac{2}{7}$ d. $d = 1$

e. $r = 22$ f. $r = 0.125$ g. $r = 8\frac{1}{2}$ h. $r = \frac{1}{10}$

12. a. 16π b. 66.8π c. 0.14π d. 178π

e. 26π f. π g. 200π h. 155π

13. a. 81π b. $10{,}000\pi$ c. 0.09π d. 12.25π

e. 100π f. 6.25π g. π h. 0.0064π

14. 198 **15**. 748 **16**. $P = 100$; $A = 625$ **17**. π

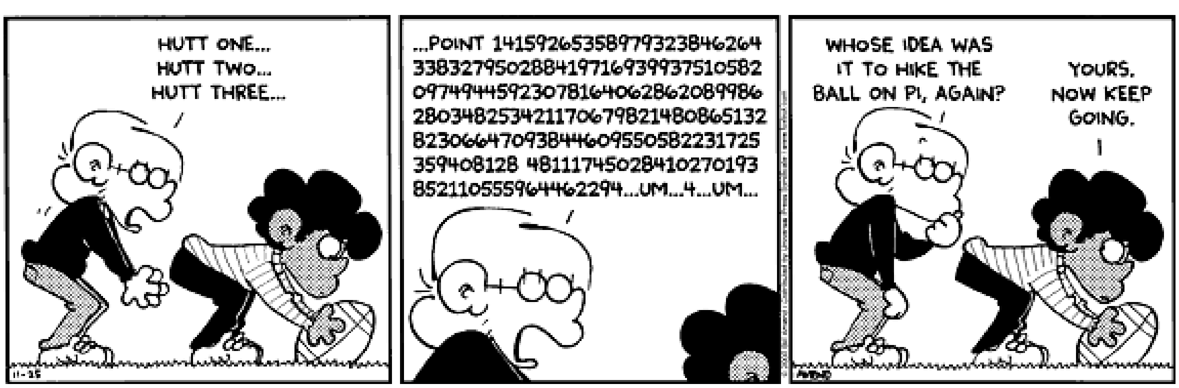
18. 30 **19**. 44 **20**. 49π **21**. 24π

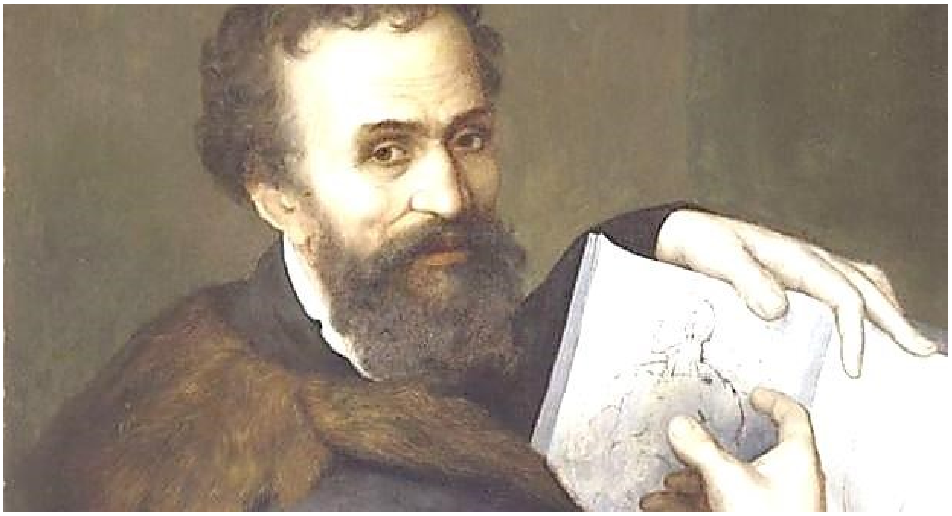
❑ *TO ∞ AND BEYOND*

A. Which is a better approximation of π: 3.14 or $\frac{22}{7}$?

B. A circle has an area of $56{,}987\pi^5$ square kilometers. Find the exact *ratio* of its diameter to its circumference.

C. "*A 20-inch-diameter pizza contains* <u>*twice*</u> *as much pizza as a 10-inch-diameter pizza.*" Any comments on this statement?





"I am still learning."

– **Michelangelo, at age 87**