

---

---

# GEOMETRY:

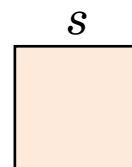
## SQUARES AND CIRCLES

---

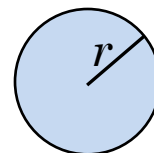
---

### □ REVIEW OF FORMULAS

If  $s$  is the length of each side of a **square**, then its perimeter is  $4s$  and its area is  $s^2$ .



If  $r$  is the radius of a **circle**, then its diameter is  $2r$ , its circumference is  $2\pi r$ , and its area is  $\pi r^2$ .



---

## Homework

---

1. True/False: Every square is a rectangle.
2. True/False: Every rectangle is a square.
3. Each side of a square is 17. Find the square's perimeter and area.
4. The radius of a circle is 10. Find the diameter, the circumference, and the area in exact form (that means leave  $\pi$  as  $\pi$ ).
5. Each side of a square is 25. Find the square's perimeter and area.
6. The radius of a circle is 1. Find the diameter, the circumference, and the area.
7. The radius of a circle is  $\frac{1}{2}$ . Find the diameter, the circumference, and the area.

**□ SQUARES**

**EXAMPLE 1:** The perimeter of a square is 94. What is the length of each side of the square?

**Solution:** The formula for the perimeter of a square is

$$P = 4s$$

Substituting the given information, we get

$$94 = 4s$$

Dividing each side of the equation by 4 gives

$$\frac{94}{4} = \frac{4s}{4}, \text{ or } \boxed{s = 23.5}$$

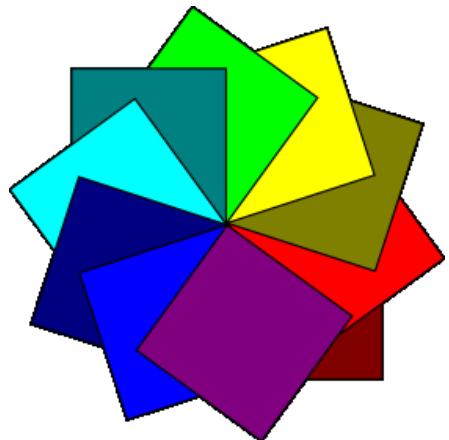
**EXAMPLE 2:** The area of a square is 361. How long is each side of the square?

**Solution:** The area of a square is given by  $A = s^2$ . Plugging 361 in for  $A$  yields the equation

$$361 = s^2$$

We need to take the square root of 361 in order to solve for  $s$ . Using a calculator (or clever guessing), we conclude that

$$\boxed{s = 19}$$



**EXAMPLE 3:**     **The perimeter of a square is 50.4. Find the area of the square.**

**Solution:**     We're being asked for the area of a square. But the area formula,  $A = s^2$ , requires that we know the side, and we don't. However, we are given the perimeter. Do you see what we're going to do? We will use the perimeter to find the side, and then use the side to find the area.

$$\begin{aligned}
 P &= 4s && \text{(perimeter of a square)} \\
 \Rightarrow 50.4 &= 4s && \text{(put in the given area)} \\
 \Rightarrow \underline{s = 12.6} &&& \text{(divide each side by 4)}
 \end{aligned}$$

Therefore,

$$\begin{aligned}
 A &= s^2 && \text{(area of a square)} \\
 &= (12.6)^2 && \text{(put in the } s \text{ just calculated)} \\
 &= 158.76 && \text{(multiply 12.6 by itself)}
 \end{aligned}$$

We conclude that the area is 158.76

**EXAMPLE 4:**     **The area of a square is 625. What is the square's perimeter?**

**Solution:**     This is the reverse of the previous example, but we solve it in essentially the same way: We'll use the given area to determine the side, and then use the side to find the perimeter.

$$A = s^2 \Rightarrow 625 = s^2 \Rightarrow \underline{s = 25}$$

And so,

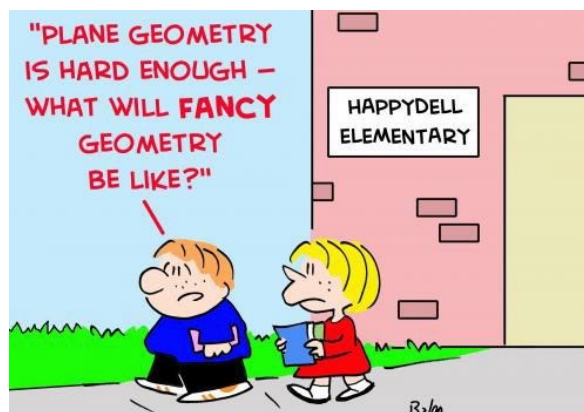
$$P = 4s = 4(25) = \span style="border: 1px solid black; padding: 2px 10px;">100$$

---

## Homework

---

8. For each problem, the perimeter of a square is given. Find the **side** of the square. You may use a calculator.
- a.  $P = 144$       b.  $P = 49.2$       c.  $P = 210$       d.  $P = 0.092$   
e.  $P = 92$       f.  $P = 10.4$       g.  $P = 504$       h.  $P = 0.04$
9. For each problem, the area of a square is given. Find the **side** of the square. You may use a calculator.
- a.  $A = 144$       b.  $A = 6.76$       c.  $A = 0.0625$       d.  $A = 6,241$   
e.  $A = 529$       f.  $A = 6.25$       g.  $A = 0.0016$       h.  $A = 1,764$
10. For each problem, the perimeter of a square is given. Find the **area** of the square. You may use a calculator.
- a.  $P = 48$       b.  $P = 50$       c.  $P = 9.6$       d.  $P = 201.2$   
e.  $P = 100$       f.  $P = 30$       g.  $P = 20.8$       h.  $P = 165.2$
11. For each problem, the area of a square is given. Find the **perimeter** of the square. You may use a calculator.
- a.  $A = 121$       b.  $A = 2116$       c.  $A = 18.49$       d.  $A = 0.09$   
e.  $A = 196$       f.  $A = 841$       g.  $A = 12.25$       h.  $A = 0.0025$



## □ CIRCLES

### Deriving the Circumference Formula: $C = 2\pi r$

The next problem we're going to solve using our algebra skills is to derive the formula  $C = 2\pi r$  for the circumference of a circle whose radius is  $r$ .

We begin with the definition of  $\pi$  from earlier in the class, namely that  $\pi$  is the ratio of the circumference of any circle to its diameter:

$$\pi = \frac{C}{d}$$

Multiplying each side of this equation by  $d$  gives:

$$\pi[d] = \frac{C}{d}[d]$$

Simplifying gives

$$\pi d = C$$

Now we turn the equation around and also change  $d$  into  $2r$ , since the diameter is twice the radius:

$$C = \pi(2r)$$

Applying the fact that multiplication is an associative operation, we can "shift" the parentheses to associate the  $\pi$  and the 2:

$$C = (\pi 2)r$$

Since multiplication is also a commutative operation, we can switch the  $\pi$  and the 2 to get:

$$C = (2\pi)r$$

Again, since multiplication is an associative operation, the parentheses are redundant (unnecessary) and we can remove them. Our final formula:

$C = 2\pi r$
--------------

**EXAMPLE 5:** The diameter of a circle is 12.  
Find the area.

**Solution:** The formula for the area of a circle,  $A = \pi r^2$ , requires that we have the radius, which was not explicitly given to us in the problem. But the diameter of the circle was given, so we can use this diameter to find the radius, and then use the radius to find the area:

$$r = \frac{d}{2} = \frac{12}{2} = 6$$

← given in the problem

Now we know that  $r = 6$ , so the area is calculated as follows:

$$A = \pi r^2 = \pi \cdot 6^2 = \boxed{36\pi}$$

**EXAMPLE 6:** Find the circumference of a circle whose diameter is 9.

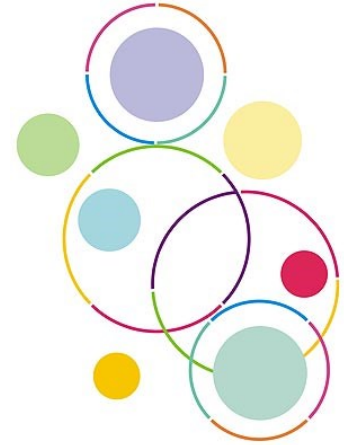
**Solution:** The process is analogous to the previous problem.

$$r = \frac{d}{2} = \frac{9}{2}, \text{ and so}$$

$$C = 2\pi r = 2\pi\left(\frac{9}{2}\right) = 2\pi\left(\frac{9}{2}\right) = \boxed{9\pi}$$

**EXAMPLE 7:** The circumference of a circle is  $24\pi$ . Find the radius.

**Solution:** The circumference of a circle is given by  $C = 2\pi r$ . Also, the circumference is given to be  $24\pi$ . So if we set the



circumference formula ( $2\pi r$ ) equal to the given circumference ( $24\pi$ ), we should be able to solve for  $r$ :

$$\begin{aligned} 2\pi r &= 24\pi && (C = 2\pi r) \\ \Rightarrow \frac{2\pi r}{2\pi} &= \frac{24\pi}{2\pi} && (\text{divide each side by } 2\pi) \\ \Rightarrow \frac{\cancel{2\pi} r}{\cancel{2\pi}} &= \frac{\overset{12}{\cancel{24}} \cancel{\pi}}{\underset{1}{\cancel{2}} \cancel{\pi}} && (\text{divide out common factors}) \\ \Rightarrow \boxed{r = 12} &&& (\text{and we've isolated the } r) \end{aligned}$$

**EXAMPLE 8:** Find the radius of a circle if it's known that its area is  $121\pi$ .

**Solution:** We set the area formula ( $\pi r^2$ ) equal to the given area ( $121\pi$ ), and solving for  $r$  will be as easy as  $\pi$ .

$$\pi r^2 = 121\pi \Rightarrow \frac{\cancel{\pi} r^2}{\cancel{\pi}} = \frac{121\cancel{\pi}}{\cancel{\pi}} \Rightarrow r^2 = 121$$

Using a calculator, or better yet, your brain, we see that  $\boxed{r = 11}$

## Homework

12. Find the circumference and the area of the circle whose diameter is given:
- a.  $d = 10$       b.  $d = 13$       c.  $d = 2$       d.  $d = 200$
13. Find the radius of the circle given its circumference or area:
- a.  $C = 30\pi$       b.  $C = 19\pi$       c.  $A = 100\pi$       d.  $A = \pi$

**EXAMPLE 9:** The circumference of a circle is  $4.6\pi$ . Find the area.

**Solution:** First ask yourself, What do I need in order to find the area? The radius, of course. But was the radius given to us? No, but the circumference was, so we can use the circumference to find the radius, and then use that radius to find the area.

Step 1: Use the circumference to find the radius

The circumference is given to be  $4.6\pi$ .

$$\Rightarrow 2\pi r = 4.6\pi \quad (\text{circumference} = 2\pi r)$$

$$\Rightarrow \frac{2\pi r}{2\pi} = \frac{4.6\pi}{2\pi} \quad (\text{divide each side by } 2\pi)$$

$$\Rightarrow r = 2.3 \quad (\text{simplify each side})$$

Step 2: Use the radius to find the area

$$A = \pi r^2 = \pi(2.3)^2 = \boxed{5.29\pi}$$



**EXAMPLE 10:** The area of a circle is  $2.25\pi$ . Find the circumference.

**Solution:** Reversing the logic in the previous problem, we use the given area to calculate the radius, and then use that radius to find the circumference.

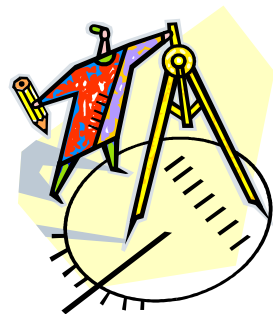
$$\pi r^2 = 2.25\pi \Rightarrow \frac{\pi r^2}{\pi} = \frac{2.25\pi}{\pi} \Rightarrow r^2 = 2.25 \Rightarrow r = 1.5$$

$$\text{It thus follows that } C = 2\pi r = 2\pi(1.5) = 2(1.5)\pi = \boxed{3\pi}$$



## Homework

14. Find the area of the circle with the given circumference:
- a.  $C = 10\pi$     b.  $C = 2\pi$     c.  $C = \pi$     d.  $C = 5\pi$
15. Find the circumference of the circle with the given area:
- a.  $A = 25\pi$     b.  $A = \pi$     c.  $A = 81\pi$     d.  $A = 225\pi$
16. a. Find the circumference of a circle with diameter 2.  
b. Find the area of a circle with diameter 30.  
c. Find the radius of a circle with circumference  $38\pi$ .  
d. Find the diameter of a circle with circumference  $8\pi$ .  
e. Find the radius of a circle with area  $576\pi$ .  
f. Find the diameter of a circle with area  $400\pi$ .  
g. Find the area of a circle with circumference  $2\pi$ .  
h. Find the circumference of a circle with area  $16\pi$ .  
i. Find the circumference of a circle with diameter 28.  
j. Find the area of a circle with diameter 28.  
k. Find the radius of a circle with circumference  $34\pi$ .  
l. Find the diameter of a circle with circumference  $20\pi$ .  
m. Find the radius of a circle with area  $529\pi$ .  
n. Find the diameter of a circle with area  $81\pi$ .  
o. Find the area of a circle with circumference  $18\pi$ .  
p. Find the circumference of a circle with area  $169\pi$ .



---

## Review Problems

---

17. The perimeter of a square is 73.6. Find the side of the square.

18. The area of a square is 10,000. Find the side of the square.

19. Find the area of a square whose perimeter is 180.

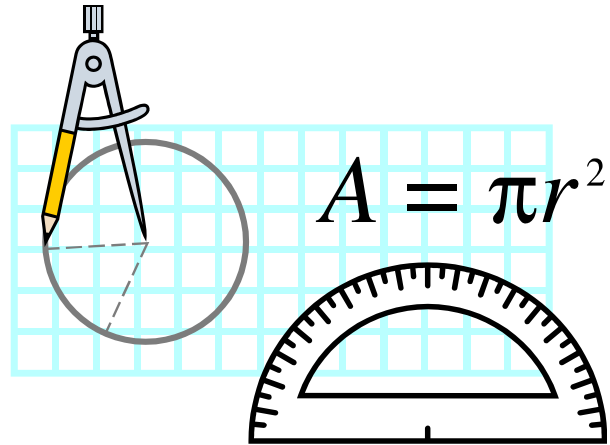
20. Find the perimeter of a square whose area is 6.25.

21. The circumference of a circle is  $20\pi$ . Find the area.

22. The area of a circle is  $49\pi$ . Find the circumference.

23. True/False:

- a. If the area of a square is 25, its perimeter is 20.
- b. If the perimeter of a square is 16, its area is 16.
- c. If the diameter of a circle is 12, its area is  $144\pi$ .
- d. If the diameter of a circle is 20, its circumference is  $20\pi$ .
- e. If the circumference of a circle is  $20\pi$ , its area is  $100\pi$ .
- f. If the area of a circle is  $49\pi$ , its diameter is 14.
- g. If the radius of a circle is 11, its area is 121.
- h. If the area of a circle is  $36\pi$ , its circumference is  $36\pi$ .



---

# Solutions

---

- 1.** True                      **2.** False                      **3.**  $P = 68; A = 289$   
**4.**  $d = 20; C = 20\pi; A = 100\pi$     **5.**  $P = 100; A = 625$   
**6.**  $d = 2; C = 2\pi; A = \pi$             **7.**  $d = 1; C = \pi; A = \frac{1}{4}\pi$   
**8.** a.  $s = 36$                   b.  $s = 12.3$                   c.  $s = 52.5$                   d.  $s = 0.023$   
          e.  $s = 23$                   f.  $s = 2.6$                   g.  $s = 126$                   h.  $s = 0.01$   
**9.** a.  $s = 12$                   b.  $s = 2.6$                   c.  $s = 0.25$                   d.  $s = 79$   
          e.  $s = 23$                   f.  $s = 2.5$                   g.  $s = 0.04$                   h.  $s = 42$   
**10.** a.  $A = 144$                   b.  $A = 156.25$                   c.  $A = 5.76$                   d.  $A = 2530.09$   
          e.  $A = 625$                   f.  $A = 56.25$                   g.  $A = 27.04$                   h.  $A = 1705.69$   
**11.** a.  $P = 44$                   b.  $P = 184$                   c.  $P = 17.2$                   d.  $P = 1.2$   
          e.  $P = 56$                   f.  $P = 116$                   g.  $P = 14$                   h.  $P = 0.2$   
**12.** a.  $C = 10\pi; A = 25\pi$                   b.  $C = 13\pi; A = 42.25\pi$   
          c.  $C = 2\pi; A = \pi$                   d.  $C = 200\pi; A = 10,000\pi$   
**13.** a.  $r = 15$                   b.  $r = 9.5$                   c.  $r = 10$                   d.  $r = 1$   
**14.** a.  $A = 25\pi$                   b.  $A = \pi$                   c.  $A = 0.25\pi$                   d.  $A = 6.25\pi$   
**15.** a.  $C = 10\pi$                   b.  $C = 2\pi$                   c.  $C = 18\pi$                   d.  $C = 30\pi$   
**16.** a.  $C = 2\pi$                   b.  $A = 225\pi$                   c.  $r = 19$                   d.  $d = 8$   
          e.  $r = 24$                   f.  $d = 40$                   g.  $A = \pi$                   h.  $C = 8\pi$   
          i.  $C = 28\pi$                   j.  $A = 196\pi$                   k.  $r = 17$                   l.  $d = 20$   
          m.  $r = 23$                   n.  $d = 18$                   o.  $A = 81\pi$                   p.  $C = 26\pi$   
**17.** 18.4                      **18.** 100                      **19.** 2025  
**20.** 10                      **21.**  $100\pi$                       **22.**  $14\pi$   
**23.** a. T                  b. T                  c. F                  d. T  
          e. T                  f. T                  g. F                  h. F

**□ TO  $\infty$  AND BEYOND**

- A. Find a square whose area is equal to its perimeter (ignoring the fact that the units of area are different from the units of perimeter).
- B. Find a rectangle (which is not a square) whose area is equal to its perimeter (ignoring the fact that the units of area are different from the units of perimeter).
- C. Find a circle whose area is equal to its circumference (even though the units must be different).
- D. Find the radius of a circle whose circumference is 20.
- E. Find the radius of a circle whose area is 10.
- F. By what factor must the radius of a circle be increased in order to increase the circumference by a factor of 4?
- G. By what factor must the radius of a circle be increased in order to increase the area by a factor of 9?

“The wisest mind  
has something  
yet to learn.”

**George Santayana (1863 - 1952)**