
GEOMETRY:

TRIANGLES AND ANGLES

□ INTRODUCTION

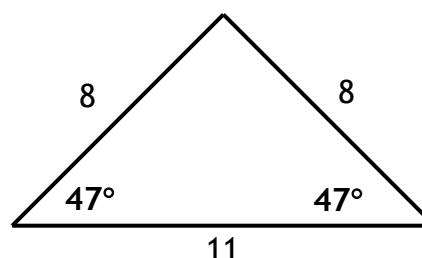
An isosceles triangle has at least two equal sides -- we know that. It's also the case that an isosceles triangle must have at least two equal angles in it, namely the angles which are opposite the equal sides.

It's a fact from geometry that the sum of the three inside angles of any triangle is always 180° . For example, in the isosceles triangle above, the top angle must be 86° , because

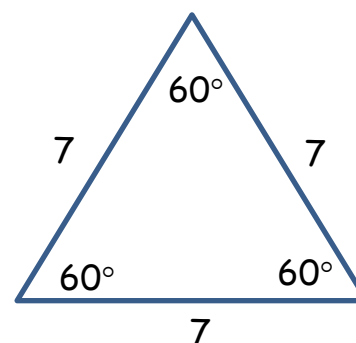
$$47^\circ + 47^\circ + 86^\circ = 180^\circ$$

Because an equilateral triangle has three equal sides, all the angles must be the same -- namely 60° each -- since

$$60^\circ + 60^\circ + 60^\circ = 180^\circ$$



An Isosceles Triangle



An Equilateral Triangle
(It's also isosceles)

Homework

1. Two of the angles in a triangle are 100° and 30° . What is the measure of the third angle?
2. Two of the angles in an isosceles triangle are 35° and 35° . Find the measure of the third angle.

3. Name the angles in any equilateral triangle.
4. True/False:
 - a. The two acute angles (the ones that are not 90°) in a right triangle must be the same.
 - b. The two acute angles in a right triangle have a sum of 90° .

□ **ANGLE PROBLEMS**

The previous chapter's triangle problems dealt with perimeter. The following problems deal with the three angles in a triangle. Notice that the problems do not specify what the sum of the three angles is -- that's because we know that **the sum of the inside angles in any triangle is 180°** (and it's your job to remember that fact!).

EXAMPLE 1: **The second angle of a triangle is 12° less than 4 times the first angle. The third angle is 30° less than the second angle. What is the measure of each angle?**

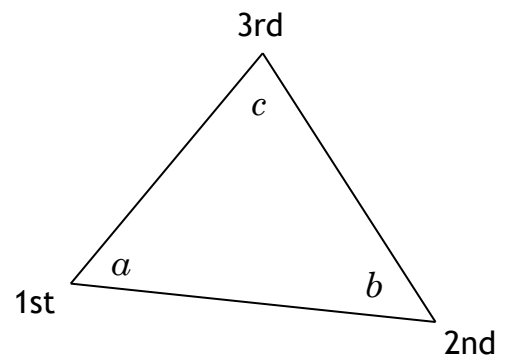
Solution: First, since the problem talks about the three angles of a triangle, the key to the equation will be that the angles must total 180° . We start by giving names to the three angles inside the triangle:

$$a = \text{1st angle}$$

$$b = \text{2nd angle}$$

$$c = \text{3rd angle}$$

Now we parse the given problem phrase by phrase:



The second angle of a triangle is 12° less than 4 times the first angle translates to

$$b = 4a - 12$$

The third angle is 30° less than the second angle becomes

$$c = b - 30$$

Our equation is

$$\begin{aligned}
 a + b + c &= 180 && \text{(angles total } 180^\circ\text{)} \\
 \Rightarrow a + \boxed{4a - 12} + \boxed{b - 30} &= 180 && \text{(substitute for } b \text{ \& } c\text{)} \\
 \Rightarrow 5a - 42 + b &= 180 && \text{(combine like terms)} \\
 \Rightarrow 5a - 42 + \boxed{4a - 12} &= 180 && \text{(substitute for } b\text{)} \\
 \Rightarrow 9a - 54 &= 180 && \text{(combine like terms)} \\
 \Rightarrow 9a &= 234 && \text{(add 54 to each side)} \\
 \Rightarrow a &= \mathbf{26} && \text{(divide by 9)}
 \end{aligned}$$

We have our final answers:

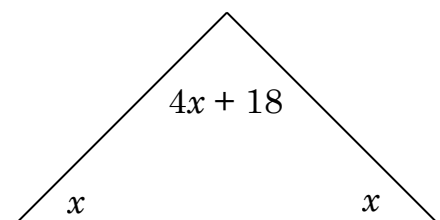
The first angle is $a = \boxed{26^\circ}$

The second angle = $b = 4a - 12 = 4(\mathbf{26}) - 12 = \boxed{92^\circ}$

The third angle = $c = b - 30 = \mathbf{92} - 30 = \boxed{62^\circ}$

EXAMPLE 2: The third angle of an isosceles triangle is 18° more than 4 times one of the equal angles. What is the measure of each angle?

Solution: For variety, let's try solving this problem using just one variable. Since two of the angles must be equal,



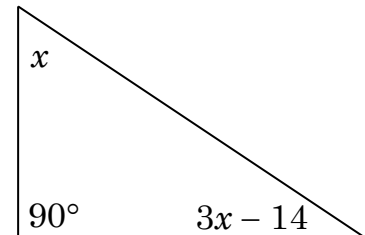
let's call each of them x . So the third angle, which is 18° more than 4 times x , can be written $4x + 18$. The sum of the three angles must be 180° :

$$\begin{aligned} x + x + (4x + 18) &= 180 && \text{(3 angles total } 180^\circ\text{)} \\ \Rightarrow x + x + 4x + 18 &= 180 && \text{(remove parentheses)} \\ \Rightarrow 6x + 18 &= 180 && \text{(combine like terms)} \\ \Rightarrow 6x &= 162 && \text{(subtract 18 from each side)} \\ \Rightarrow x &= \mathbf{27} && \text{(divide each side by 6)} \\ \Rightarrow 4x + 18 &= 4(\mathbf{27}) + 18 = 126 \end{aligned}$$

Hence, the three angles are $27^\circ, 27^\circ,$ and 126°

EXAMPLE 3: One of the acute angles of a right triangle is 14° less than 3 times the other acute angle. Find the measure of the larger acute angle.

Solution: Again, let's use just one variable to solve this problem. Since one of the angles in a right triangle is 90° , we need only concern ourselves with the two acute angles. Calling one of them x means that the other one must be called $3x - 14$. Since the sum of the three angles must be 180° , we get the equation

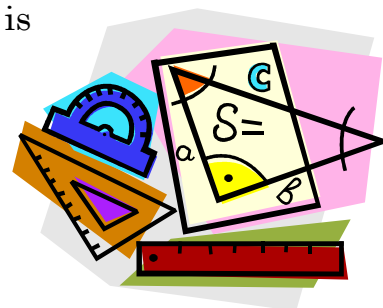


$$\begin{aligned} x + (3x - 14) + 90 &= 180 && \text{(3 angles total } 180^\circ\text{)} \\ \Rightarrow x + 3x - 14 + 90 &= 180 && \text{(drop the parentheses)} \\ \Rightarrow 4x + 76 &= 180 && \text{(combine like terms)} \\ \Rightarrow 4x &= 104 && \text{(subtract 76 from each side)} \\ \Rightarrow x &= \mathbf{26} && \text{(divide each side by 4)} \\ \Rightarrow 3x - 14 &= 3(\mathbf{26}) - 14 = 64 \end{aligned}$$

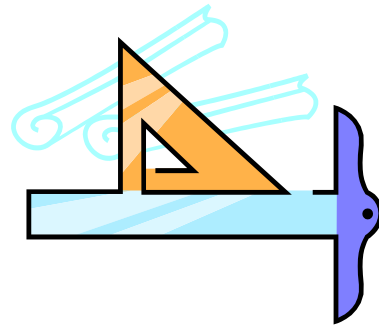
Therefore, the larger acute angle is 64°

Homework

5. One of the acute angles of a right triangle is 18° more than 2 times the other acute angle. Find the measures of the two acute angles.
6. The third angle of an isosceles triangle is 15° more than 3 times one of the equal angles. What is the measure of each angle?
7. The second angle of a triangle is 16° more than 3 times the first angle. The third angle is 125° less than the second angle. What is the measure of each angle?
8. The third angle of an isosceles triangle is 4° less than 2 times one of the equal angles. What is the measure of each angle?
9. The second angle of a triangle is 38° less than 4 times the first angle. The third angle is 13° more than the second angle. What is the measure of each angle?
10. One of the acute angles of a right triangle is 6° less than 5 times the other acute angle. Find the measures of the two acute angles.
11. One of the acute angles of a right triangle is 10° less than 3 times the other acute angle. Find the measures of the two acute angles.
12. The third angle of an isosceles triangle is 8° less than 2 times one of the equal angles. What is the measure of each angle?
13. The second angle of a given triangle is 66° more than its first angle, while its third angle is 18° less than 4 times the first angle. Find the measure of each angle.



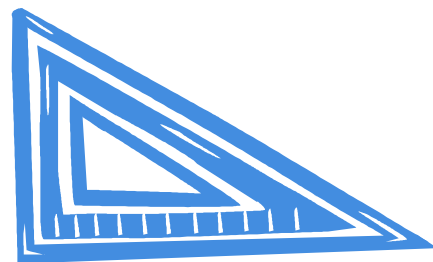
14. One of the acute angles of a right triangle is 18° less than 2 times the other acute angle. Find the measures of the two acute angles.
15. One of the acute angles of a right triangle is 60° less than 2 times the other acute angle. Find the measures of the two acute angles.
16. The third angle of an isosceles triangle is 68° less than 2 times one of the equal angles. What is the measure of each angle?
17. The second angle of a triangle is 20° less than the first angle. The third angle is 20° more than twice the second angle. What is the measure of each angle?



Review Problems

18. The second angle of a triangle is 30° more than the first angle, while the first angle is 26° more than 3 times the third angle. What is the measure of each angle?
19. One of the acute angles of a right triangle is 18° more than 5 times the other acute angle. What is the measure of each acute angle?
20. The first angle of a triangle is 1° more than 8 times the third angle, while the second angle is 47° more than 3 times the third angle. What is the measure of each angle?
21. The third angle of a triangle is 90° less than 10 times the second angle, while the first angle is 50° less than 9 times the second angle. What is the measure of each angle?
22. One of the acute angles of a right triangle is 10° less than the other acute angle. What is the measure of each acute angle?

23. The second angle of a triangle is 96° less than 3 times the third angle, while the first angle is 25° less than 3 times the third angle. What is the measure of each angle?
24. The third angle of a triangle is 38° more than 3 times the second angle, while the first angle is 43° less than the third angle. What is the measure of each angle?
25. The second angle of a triangle is 11° more than 2 times the third angle, while the first angle is 55° less than 5 times the third angle. What is the measure of each angle?
26. One of the acute angles of a right triangle is 14° less than the other acute angle. What is the measure of each acute angle?
27. The second angle of a triangle is 3° less than 4 times the first angle, while the third angle is 42° more than the second angle. What is the measure of each angle?
28. The second angle of a triangle is 32° less than 6 times the first angle, while the third angle is 42° less than the second angle. What is the measure of each angle?
29. The third angle of an isosceles triangle is 12° more than 5 times one of the equal angles. What is the measure of each angle?
30. The second angle of a triangle is 28° more than 5 times the first angle, while the third angle is 42° more than 4 times the first angle. What is the measure of each angle?
31. The second angle of a triangle is 75° more than 5 times the first angle, while the third angle is 21° less than 3 times the first angle. What is the measure of each angle?
32. The third angle of an isosceles triangle is 4° more than 20 times one of the equal angles. What is the measure of each angle?
33. The second angle of a triangle is 29° more than the first angle, while the first angle is 15° more than 5 times the third angle. What is the measure of each angle?



Solutions

- | | | |
|-------------------------------------|-------------------------------------|-------------------------------------|
| 1. 50° | 2. 110° | 3. $60^\circ, 60^\circ, 60^\circ$ |
| 4. a. F b. T | 5. $24^\circ, 66^\circ$ | 6. $33^\circ, 33^\circ, 114^\circ$ |
| 7. $39^\circ, 133^\circ, 8^\circ$ | 8. $46^\circ, 46^\circ, 88^\circ$ | 9. $27^\circ, 70^\circ, 83^\circ$ |
| 10. $16^\circ, 74^\circ$ | 11. $25^\circ, 65^\circ$ | 12. $47^\circ, 47^\circ, 86^\circ$ |
| 13. $22^\circ, 70^\circ, 88^\circ$ | 14. $36^\circ, 54^\circ$ | 15. $40^\circ, 50^\circ$ |
| 16. $56^\circ, 62^\circ, 62^\circ$ | 17. $35^\circ, 55^\circ, 90^\circ$ | 18. $14^\circ, 68^\circ, 98^\circ$ |
| 19. $12^\circ, 78^\circ$ | 20. $11^\circ, 89^\circ, 80^\circ$ | 21. $16^\circ, 70^\circ, 94^\circ$ |
| 22. $50^\circ, 40^\circ$ | 23. $43^\circ, 33^\circ, 104^\circ$ | 24. $21^\circ, 101^\circ, 58^\circ$ |
| 25. $28^\circ, 67^\circ, 85^\circ$ | 26. $52^\circ, 38^\circ$ | 27. $16^\circ, 61^\circ, 103^\circ$ |
| 28. $22^\circ, 100^\circ, 58^\circ$ | 29. $24^\circ, 24^\circ, 132^\circ$ | 30. $11^\circ, 83^\circ, 86^\circ$ |
| 31. $14^\circ, 145^\circ, 21^\circ$ | 32. $8^\circ, 8^\circ, 164^\circ$ | 33. $11^\circ, 70^\circ, 99^\circ$ |

“The beginning of knowledge is the discovery of something we do not understand.”

Frank Herbert