PERCENT MIXTURE PROBLEMS, PART I

❑ *INTRODUCTION*

NaOCl
sodium hypochlorite

Suppose you have 12 quarts of a bleach solution which is at a 25% concentration. This means that 25% of the solution is bleach, while the other 75% is some neutral substance (usually water). How much of the 12 quarts is actually bleach? Since the solution is 25% bleach, we take 25% of 12, which is $12 \times 0.25 = 3$ quarts of bleach.

The 12 quarts of solution is called the total **quantity**.

The 25% is called the **concentration**.

The 3 quarts of bleach is called the actual **amount**.

The calculation above shows us the formula which ties all these ideas together, and is the basis for this chapter and the next one:

$$\text{Quantity} \times \%\ \text{Concentration} = \text{Amount}$$

Two Important Notes:

1. To convert 63% to a decimal, move the decimal point (it's after the 3) two places to the left and drop the percent sign, giving 0.63. To convert the decimal 0.08 to a percent, move the decimal point two places to the right and add a percent sign to get 8%.

Homework

1. A 40-quart solution of sulfuric acid is at a 30% concentration. How many quarts of the solution are sulfuric acid? How many quarts are water?
2. A 25-kg solution of salt water is 20% NaCl (salt). How many kg of the solution consist of NaCl? How many kg are water?

❑ *EXAMPLES*

EXAMPLE 1: **A restaurant mixes 4 mL of one 19% sugar beverage with an unknown amount of an 84% sugar beverage. If the final mixture is 64% sugar, how many mL of the 84% sugar beverage were used?**

Solution: Let's organize all the known quantities and the unknown quantity in a chart for easy reference. Notice that the items being mixed (the ingredients) are in the first two rows, the final solution (the mixture) is in the bottom row, and the formula we developed above constitutes the column headings of columns 2, 3, and 4. Carefully re-read the problem and note how all of its information has been placed in the chart:

	Quantity x	Concentration =	Amount
19% sugar	4	19%	4(0.19)
84% sugar	x	84%	$0.84x$
final mixture	$x + 4$	64%	$0.64(x + 4)$

We're letting x represent the quantity of the 84% sugar beverage. Note that if we mix 4 mL with x mL, we'll get a total quantity of $x + 4$ mL.

The Quantity column provides us with a potential equation:

$4 + x = x + 4$. Unfortunately, this equation falls apart and tells us nothing about x. So we hop over to the Amount column and use the following reasoning: The actual amount of sugar in the final mixture must equal the sum of the actual amounts of sugar in the ingredients:

$$4(0.19) + 0.84x = 0.64(x + 4)$$

$\Rightarrow$ $0.76 + 0.84x = 0.64x + 2.56$ (distribute)

$\Rightarrow$ $0.2x + 0.76 = 2.56$ (subtract $0.64x$)

$\Rightarrow$ $0.2x = 1.8$ (subtract 0.76)

$\Rightarrow$ $x = 9$ (divide by 0.2)

Therefore, 9 mL of the 84% sugar beverage were used.

EXAMPLE 2: **A 72% acid mixture is created by mixing 12 pints of a 66% acid solution with 8 pints of another acid solution. Find the percent concentration of the other acid solution.**

Solution: Since we are now experts on this subject, we'll let C stand for the unknown concentration and get right to the following chart:

	Quantity x	Concentration =	Amount
66% acid	12	66%	12(0.66)
unknown acid	8	C	$8C$
final mixture	20	72%	20(0.72)

We could write an equation stating that the Quantities must add up, giving us $12 + 8 = 20$ -- which is true, but useless. As in the

previous example, the Amount column gives us our equation with the variable in it:

$$12(0.66) + 8C = 20(0.72)$$

$\Rightarrow$ $7.92 + 8C = 14.4$ (arithmetic)

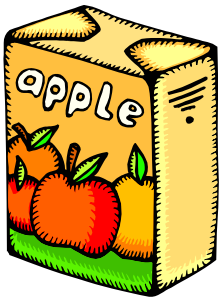
$\Rightarrow$ $8C = 6.48$ (subtract 7.92)

$\Rightarrow$ $C = 0.81$ (divide by 8)

Convert the decimal 0.81 to a percent by moving the decimal point two places to the right.

Thus, the concentration of the other acid is $\boxed{81\%}$

EXAMPLE 3: **A grocer mixes 2 L of a 34% apple juice beverage with 4 L of a 19% apple juice beverage. What is the percent concentration of apple juice in the resulting mixture?**



Solution: This example is essentially the same as the previous one, except that this problem asks for the concentration of the final mixture, not one of the ingredients.

	Quantity	x Concentration	= Amount
34% juice	2	34%	2(0.34)
19% juice	4	19%	4(0.19)
final mixture	6	C	$6C$

Although the Quantity column gives us a true equation, $2 + 4 = 6$, this equation is useless in trying to solve for C. So let's jump to the Amount column and use the concept that the actual amounts of apple juice in the 34% and the 19% juices must add up to the actual amount of apple juice in the mixture:

$$2(0.34) + 4(0.19) = 6C$$

$$\Rightarrow \quad 0.68 + 0.76 = 6C$$

$$\Rightarrow \quad 1.44 = 6C$$

$$\Rightarrow \quad C = 0.24$$

Hence, the concentration of the mixture is $\boxed{24\%}$

Homework

3. A chemist mixes 8 mL of a 37% glycerin solution with some 57% glycerin solution. If the final mixture is 49% glycerin, how many mL of the 57% glycerin solution were used?

4. A 63% acid solution is created by mixing 3 liters of an 87% acid solution with 12 liters of another acid solution. Find the concentration of the other acid solution.

5. An allergist wants to mix some pure water with 2 ounces of an 18% pollen serum to produce a serum that is 12% pollen. How many ounces of pure water should he use?

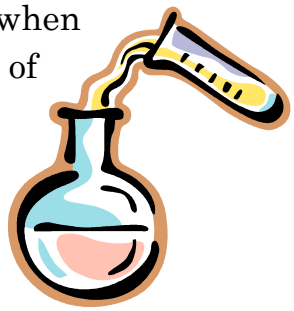
6. A restaurant mixes 5 liters of a 13% sugar liquid with some 21% sugar liquid. If the final mixture is 19% sugar, how many liters of the 21% sugar liquid were used?

7. An 18-gallon solution that is 45% vinegar is mixed with 9 gallons of another solution. The resulting solution is 58% vinegar. Find the percentage of vinegar in the other solution.

8. A researcher has a 4-L mixture that is 40% saline. How many liters of pure water should she add to make a mixture that is 16% saline?

9. A company mixes 14 L of pure sulfate with 16 L of 10% sulfate shampoo. What percentage sulfate is the resulting mixture?

10. What is the percent concentration of sand in a mixture a mason made by mixing 12 pounds of pure sand with 2 pounds of 72% sand cement?
11. How many mL of pure water should a detective mix with 2 mL of a 54% cyanide poison to produce a poison that is 12% cyanide?
12. A bakery mixes 8 pounds of a 64% butter frosting with 4 pounds of a 73% butter frosting. What is the percentage of butter in the resulting mixture?
13. A 52% mixture is created by mixing 16 liters of a 97% solution with 24 liters of another solution. Find the concentration of the other solution.
14. What is the concentration of the mixture formed when 57 liters of a 30% solution are mixed with 3 liters of a 50% solution?
15. A 22% mixture is created by mixing 22 liters of a 5% solution with 17 liters of another solution. Find the concentration of the other solution.
16. What is the concentration of the mixture formed when 4 liters of a 43% solution are mixed with 60 liters of an 11% solution?



17. A 71% mixture is created by mixing 56 liters of a 98% solution with 54 liters of another solution. Find the concentration of the other solution.
18. What is the concentration of the mixture formed when 28 liters of a 90% solution are mixed with 36 liters of a 42% solution?
19. How many liters of a 62% solution must be added to 60 liters of a 77% solution to get a mixture whose concentration is 72%?
20. How many liters of a 99% solution must be added to 17 liters of a 96% solution to get a mixture whose concentration is 98%?

Solutions

1. 12 qts acid; 28 qts H_2O

2. 5 kg NaCl; 20 kg H_2O

3. 12 mL

4. 57% acid

5. 1 oz

6. 15 L

7. 84% vinegar

8. 6 L

9. 52% sulfate

10. 96% sand

11. 7 mL

12. 67% butter

13. 22%

14. 31%

15. 44%

16. 13%

17. 43%

18. 63%

19. 30 L

20. 34 L

"Few will have the greatness to bend history itself; but each of us can work to change a small portion of events, and in the total of all those acts will be written the history of this generation." Robert F. Kennedy