MOTION PROBLEMS: OPPOSITE DIRECTION

Mike and Sarah start from the burger stand and skate in opposite directions. Mike's speed is 5 less than 3 times Sarah's speed. In 10 hours they are 70 miles apart. Find the speed of both skaters.

Solution: Let's organize all the information in a table using the basic $rt = d$ formula we're learning in this chapter. Down the first column are the names of our two skaters. Across the first row are the three components of motion, the rate (speed), the time, and the distance. We've written the formula to help us remember the basic relationship among these three concepts.

	Rate × **Time** = **Distance**		
Mike			
Sarah			

Since each skater's speed is being asked for (they're the unknowns), we'll let M stand for Mike's speed and S stand for Sarah's speed, and so these variables go into the Rate column.

As for the Time column, the problem states that each skater skated for exactly 10 hours, so each of their travel times is 10.

Since Distance = Rate × Time, the Distance column is simply the product of the Rate and Time columns for both Mike and Sarah.

	Rate × Time = Distance		
Mike	M	10	$10M$
Sarah	S	10	$10S$

Since there are two unknowns in this problem, it's likely we'll need two equations. Let's look at the rates first: From the phrase in the problem "Mike's speed is 5 less than 3 times Sarah's speed" we create the equation

$$M = 3S - 5 \qquad \text{[Equation 1]}$$

To determine the second equation, we have to picture where the skaters are going. They start in the same place and then proceed to skate in opposite directions and end up 70 miles from each other. Therefore, the sum of their individual distances must be 70. Well, Mike skated a distance of $10M$ miles while Sarah went $10S$ miles. So 70 must be the sum of $10M$ and $10S$:

$$10M + 10S = 70 \qquad \text{[Equation 2]}$$

Now substitute Equation 1 into Equation 2:

$10(\mathbf{3S - 5}) + 10S = 70$ (replaced M with $3S - 5$)

$\Rightarrow \quad 30S - 50 + 10S = 70$ (distribute)

$\Rightarrow \quad 40S - 50 = 70$ (combine like terms)

$\Rightarrow \quad 40S = 120$ (add 50 to each side)

$\Rightarrow \quad \underline{S = 3}$ (divide each side by 40)

Recall that S stood for Sarah's speed, so we know for sure that Sarah skated 3 mph. To find Mike's speed we use Equation 1 and Sarah's speed:

$$\begin{aligned} M &= 3S - 5 \\ &= 3(\mathbf{3}) - 5 \\ &= 9 - 5 \\ &= \underline{4} \end{aligned}$$

Note: We could have used the **Addition Method** to solve the system of equations:

$$M = 3S - 5$$
$$10M + 10S = 70$$

This shows that Mike skated at a rate of 4 mph. We now have the complete answer to the question:

> Mike's speed was 4 mph and
>
> Sarah's speed was 3 mph.

Homework

1. Two pedestrians leave the same place and walk in opposite directions. The speed of one of the pedestrians is 5 mph less than 7 times the other. In 6 hours they are 354 miles apart. Find the speed of each pedestrian.
2. Two skaters leave the same place and skate in opposite directions. The speed of one of the skaters is 8 mph less than 10 times the other. In 9 hours they are 819 miles apart. Find the speed of each skater.
3. Two joggers leave the same place and jog in opposite directions. The speed of one of the joggers is 9 mph more than 5 times the other. In 7 hours they are 357 miles apart. Find the speed of each jogger.
4. Two pedestrians leave the same place and walk in opposite directions. The speed of one of the pedestrians is 7 mph less than 9 times the other. In 10 hours they are 930 miles apart. Find the speed of each pedestrian.
5. Two skaters leave the same place and skate in opposite directions. The speed of one of the skaters is 1 mph more than 7 times the other. In 9 hours they are 513 miles apart. Find the speed of each skater.

Solutions

1. 8 mph & 51 mph

2. 9 mph & 82 mph

3. 7 mph & 44 mph

4. 10 mph & 83 mph

5. 7 mph & 50 mph