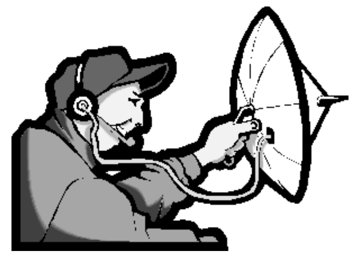
GRAPHING PARABOLAS

❑ *INTRODUCTION*



The **parabola** (accent on the *rab*) is a very special shape used in searchlights and satellite dishes. Even football sports reporters use parabolic reflectors to listen in on comments made by coaches on the sidelines and players in the huddle. In fact, when a football is thrown or punted, its path is that of a parabola.

❑ *GRAPHING A PARABOLA*

EXAMPLE 1: **Graph:** $y = x^2 - 4x + 3$

Solution: Who's to say that the graph of this formula isn't a straight line? Let's work it out; we'll find some points on our graph by choosing some values of x, and then calculate the corresponding y-values -- and we'll see what points we get.

If $x = -1$, then $y = (\mathbf{-1})^2 - 4(\mathbf{-1}) + 3 = 1 + 4 + 3 = 8 \Rightarrow$ **(–1, 8)**

If $x = 0$, then $y = (\mathbf{0})^2 - 4(\mathbf{0}) + 3 = 0 - 0 + 3 = 3 \Rightarrow$ **(0, 3)**

If $x = 1$, then $y = (\mathbf{1})^2 - 4(\mathbf{1}) + 3 = 1 - 4 + 3 = 0 \Rightarrow$ **(1, 0)**

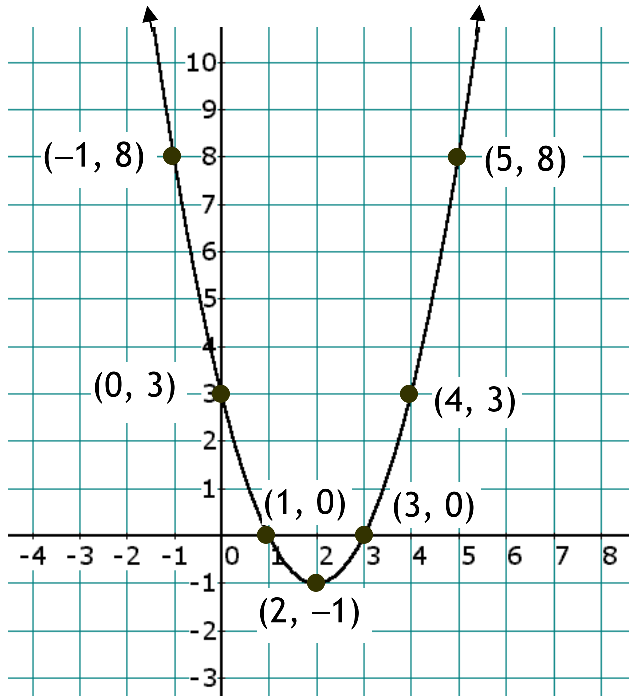
If $x = 2$, then $y = (\mathbf{2})^2 - 4(\mathbf{2}) + 3 = 4 - 8 + 3 = -1 \Rightarrow$ **(2, –1)**

If $x = 3$, then $y = (\mathbf{3})^2 - 4(\mathbf{3}) + 3 = 9 - 12 + 3 = 0 \Rightarrow$ **(3, 0)**

If $x = 4$, then $y = (\mathbf{4})^2 - 4(\mathbf{4}) + 3 = 16 - 16 + 3 = 3 \Rightarrow$ **(4, 3)**

If $x = 5$, then $y = (\mathbf{5})^2 - 4(\mathbf{5}) + 3 = 25 - 20 + 3 = 8 \Rightarrow$ **(5, 8)**

Plotting these seven points leads us to the following graph:



What do we notice about the graph? It's a curve, not a straight line. We notice that x can be any real number (but notice that y never goes below –1). Also note that the graph has one y-intercept but two x-intercepts. In addition, there is no highest point on the parabola, but the lowest point on the parabola is (2, –1), and we call this point the ***vertex*** of the parabola.

This is the shape called the ***parabola***. We say that the parabola just graphed "**opens up**." The equation of a parabola is characterized by the fact that one variable (the x) is *squared* while the other variable (the y) is raised to the first power.

EXAMPLE 2: **Graph:** $y = -x^2 - 2x - 1$

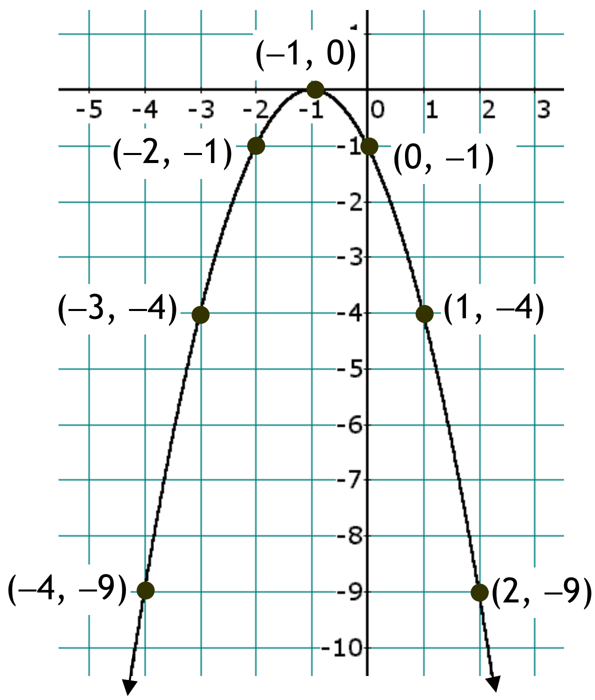
Solution: First we notice that the quadratic term, the $-x^2$, has a leading negative sign. And we remember that, due to the Order of Operations, exponents have a higher priority than minus signs, so we know that to evaluate $-x^2$, we square the x first, and apply the minus sign <u>second</u>. We'll calculate together two points on the parabola and leave the rest of the points for you to do.

If $x = 2$, then $y = -(\mathbf{2})^2 - 2(\mathbf{2}) - 1 = -4 - 4 - 1 = -9$. Thus, the point **(2, –9)** is on the graph of our parabola.

If $x = -3$, then $y = -(\mathbf{-3})^2 - 2(\mathbf{-3}) - 1 = -9 + 6 - 1 = -4$. Therefore, the point **(–3, –4)** is on the graph.

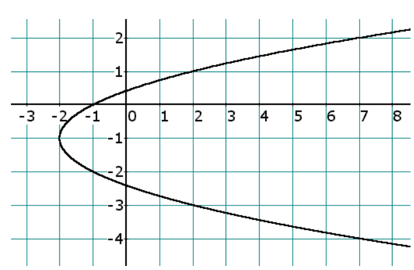
You should now do the calculations to show that each of the following points is also on the graph:

(–4, –9) **(–2, –1)** **(–1, 0)** **(0, –1)** **(1, –4)**



From the formula (and kind of from the graph) we note that *x* can be <u>any</u> real number, but that *y* never gets above 0. As for intercepts, there's one *x*-intercept and one *y*-intercept. There is no lowest point on the graph, but the highest point, the **vertex**, is the point (–1, 0). We say that this parabola **opens down**.

There are also "sideways" parabolas, but in this chapter only parabolas which open up or down will be discussed.



Homework

1. In Example 1 we saw that the graph of $y = x^2 - 4x + 3$ is a parabola opening up. Example 2 showed us that the graph of $y = -x^2 - 2x - 1$ is a parabola opening down. Take a guess what property of these equations determines whether the parabola opens up or down.

2. True/False: (Recall that all parabolas in this chapter open up or down.)

   a. Every parabola has a y-intercept.

   b. Every parabola has an x-intercept.

   c. Every parabola has a vertex.

   d. The vertex of a parabola is always the highest point of the parabola.

   e. The vertex of a parabola is always the lowest point of the parabola.

3. Graph each parabola by plotting points. Then use your graph to determine the intercepts and the vertex of your parabola:

   a. $y = 9 - x^2$

   b. $y = x^2 + 6x + 5$

   c. $y = x^2 + 2x - 8$

   d. $y = -x^2 + 4x - 4$

   e. $y = 0.5x^2$

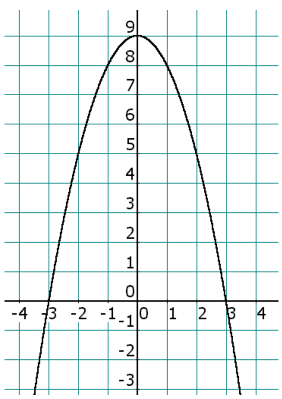
   f. $y = -0.2x^2 + 5$

Solutions

1. The coefficient of the quadratic term in Example 1 is positive, while that of the quadratic term in Example 2 is negative. That's the clue which determines whether a parabola opens up or down. Therefore, the parabola $y = \pi x^2 - 13x + 2$ opens up, whereas the parabola $y = -0.7x^2 + 99x + 14$ opens down.

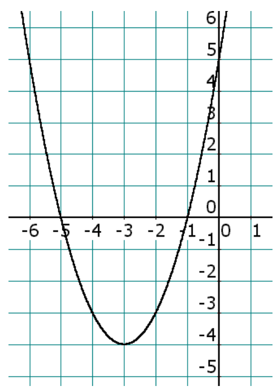
2. a. True b. False c. True d. False e. False

3. a.



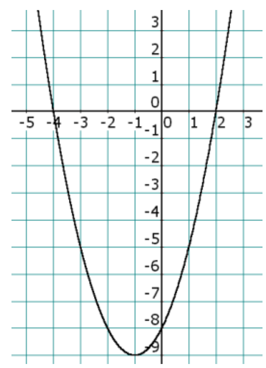
Intercepts:
(–3, 0), (3, 0), (0, 9)
Vertex: (0, 9)

b.



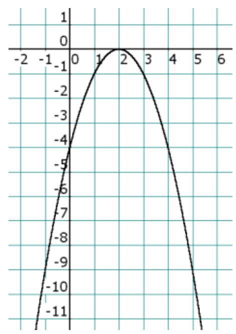
Intercepts:
(–5, 0), (–1, 0), (0, 5)
Vertex: (–3, –4)

c.



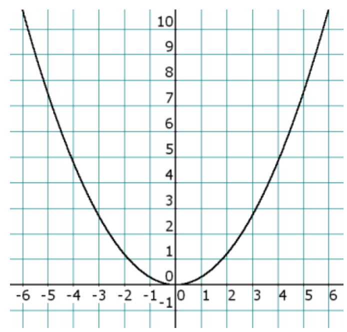
Intercepts:
(–4, 0), (2, 0), (0, –8)
Vertex: (–1, –9)

d.



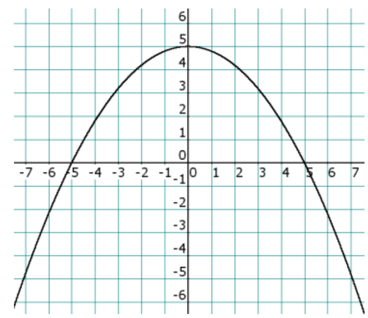
Intercepts:
(2, 0), (0, –4)
Vertex: (2, 0)

e.



Intercepts: (0, 0)
Vertex: (0, 0)

f.



Intercepts:
(–5, 0), (5, 0), (0, 5)
Vertex: (0, 5)

"It is our choices that show
what we truly are,
far more than our abilities."

– J.K. Rowling