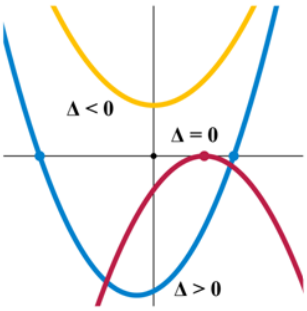
POLYNOMIALS

❑ *INTRODUCTION*



It's very difficult to define what a *polynomial* is at this point in your algebra studies, because we haven't come across many things that aren't polynomials. Suffice it to say that a typical polynomial expression looks like

$$3x^5 - \pi x^3 + x^2 - 9x + 0.8 \qquad \text{[Note: 5 terms]}$$

The main theme of a polynomial is that all of the exponents on the x (or whatever variable) must be one of the whole numbers 0, 1, 2, 3, …. The following are <u>not</u> polynomials: $8x^{-2}$ and $3x^{1/4}$, because the exponents –2 and 1/4 are not whole numbers.

❑ *WORKING WITH MONOMIALS*

A polynomial with one term, such as $7x^3$, is called a ***monomial***. Let's look at a couple of examples of multiplying monomials together, a skill we'll need when we multiply binomials together (Can you guess what a <u>bi</u>nomial is?). The key to multiplying monomials is that each monomial is a single term whose final operation is multiplication.

For example, to find the product of $7x$ and $9x$, we proceed the long way -- you don't ever actually do it this way, but it's important to see:

	$(7x)(9x)$	(the original expression)
=	$(7 \cdot x) \cdot (9 \cdot x)$	(it's all multiplication)
=	$(7 \cdot 9) \cdot (x \cdot x)$	(rearrange the factors)
=	$63 \cdot x^2$	(something times itself is squaring)
=	$\mathbf{63x^2}$	(remove the dot)

Another example is $3(-10n) = (3)(-10)n = \mathbf{-30n}$.

And don't forget that adding and subtracting don't follow the same rules as multiplication. Two monomials can be added or subtracted only if they are like terms. See if the homework sorts all of this out.

Homework

1. Simplify each expression:

a. $3(7L)$	b. $-5(2x)$	c. $-6(-2T)$
d. $20(-3w)$	e. $3 + 7L$	f. $-5 + 2x$
g. $-6 - 2T$	h. $20 - 3w$	i. $(7y)5$
j. $(-2p)(-5)$	k. $(-3a)(10)$	l. $(5n)(-2)$
m. $7y + 5$	n. $(4x)(3x)$	o. $4x + 3x$
p. $(2n)(-3n)$	q. $2n - 3n$	r. $(-8x)(-7x)$
s. $(7u)(-u)$	t. $(-4c)(4c)$	u. $-4c + 4c$
v. $(7m)(6n)$	w. $7m - 6n$	x. $(13k)(-13k)$
y. $13k - 13k$	z. $-14x + 20x$	

2. Find the **product** of

a. $3x$ and 4	b. $7y$ and 5	c. $-3n$ and 2
d. $2n$ and -10	e. 6 and 7	f. -8 and -8
g. z and z	h. $2n$ and $3n$	i. $7m$ and $-2m$
j. $2p$ and $-2p$	k. $3a$ and $-5a$	l. $-3a$ and -7
m. -10 and 10	n. $-8g$ and -10	o. 4 and $-12x$

❑ *More Like Terms*

To simplify the 6-term polynomial

$$5x^2 + 7x - 8 - 2x^2 - 9x - 20$$

we can rearrange the terms like this (since addition is commutative):

$$5x^2 - 2x^2 + 7x - 9x - 8 - 20$$ (This step is optional)

and then combine like terms to reach a result of 3 terms:

$$\mathbf{3x^2 - 2x - 28}$$ ⟵ This is called a **trinomial**.

For a second example, let's simplify an expression containing parentheses:

$$3(y^2 - 8y + 9) - (3y^2 - 7y + 1)$$ (This polynomial has 2 terms)

This next step is optional, but use it if you need to:

$$3(y^2 - 8y + 9) - \mathbf{1}(3y^2 - 7y + 1)$$

Now distribute:

$$3y^2 - 24y + 27 - 3y^2 + 7y - 1$$

Rearrange the terms into pairs of like terms:

$$3y^2 - 3y^2 - 24y + 7y + 27 - 1$$ (This step is optional)

And then combine them:

$$\mathbf{-17y + 26}$$ ⟵ This is called a **binomial**.

Homework

3. Suppose a friend believes that $4n^2$ and $7n$ are like terms, and that their sum should be $11n^3$. Prove your friend wrong by letting $n = 2$.

4. Simplify each expression by combining like terms:

a. $3x^2 - 7x + 5x^2 + 9$

b. $n^2 - 9 + 9 - n^2$

c. $1 - 3u - u^2 - 3u^2 + 7u - 1$

d. $7a^2 - 8a + 7 - 9a^2 + 7a - 7$

e. $x^2 - 3x - 1 + 7x^2 - 3x + 1$

f. $3y^2 - 2 + 3y^2 - 2$

g. $1 - 3x - x^2 + 5 - 7x + x^2$

h. $-5w^2 + 2 - 3w + 8w - 2 - w^2$

5. Simplify each expression by distributing and then combining like terms:

a. $(3c^2 - 2c - 1) + 2(c^2 + 5c - 7)$

b. $3(x^2 - 8x + 1) - 5(2x^2 + 7x - 1)$

c. $-(a^2 - a - 1) + 3(-a^2 + a)$

d. $7w^2 - 13w + 8 - (5w^2 - 3w - 2)$

e. $-(7u^2 - 7u - 6) - (-6u^2 + 3u + 5)$

f. $(3x^2 - x - 1) - (3x^2 - x - 1)$

g. $-2(x^2 - 3x + 7) - (3x^2 + 10x - 1)$

h. $-(3n^2 + 8n - 1) - 3(n^2 + 2n - 1)$

❑ *MORE DISTRIBUTING*

A polynomial with one term is called a ***monomial***, and a polynomial with two terms is called a ***binomial***. A typical problem where we must multiply a monomial by a binomial is the following:

$3x(2x + 10)$ ($3x$ is the monomial; $2x + 10$ is the binomial)

Finding the product of these two polynomials is pretty easy -- just distribute:

$3x(2x + 10)$

$= (3x)(2x) + (3x)(10)$

$= \mathbf{6x^2 + 30x}$, and it's done

Homework

6. Simplify each expression by distributing:

a. $9a(a + 9)$ b. $10x(3x - 1)$ c. $-5y(1 - y)$

d. $n(5n + 1)$ e. $5a(5a + 5)$ f. $3u(7u - 8)$

g. $-5m(m + 1)$ h. $-3t(10t + 12)$ i. $-(6x^2 - 9)$

Review Problems

7. Simplify each expression:

a. $4(3n)$ b. $-8(3x)$ c. $2b + 3b$

d. $(8y)(-2y)$ e. $8y - 2y$ f. $(2x)(-3z)$

8. Find the product of

a. $7x$ and 4 b. $3a$ and $5a$ c. $2n$ and $3m$

d. 7 and -7 e. 3 and $-13t$ f. $7x$ and $-7x$

9. Simplify each expression:

a. $7x^2 - 3x + 7 - 7x^2 - 3x - 7$ b. $-8(3y^2 - 4y - 1)$

c. $2(a^2 - 8) - (a^2 - 2a - 1)$ d. $-(4n^2 - 4n) - (4n - 4n^2)$

e. $3(4g^2 - g + 3) - 2(6g^2 + g - 1)$ f. $-8x(-7x - 5)$

Solutions

1. a. $21L$ b. $-10x$ c. $12T$ d. $-60w$ e. As is f. As is
g. As is h. As is i. $35y$ j. $10p$ k. $-30a$ l. $-10n$
m. As is n. $12x^2$ o. $7x$ p. $-6n^2$ q. $-n$ r. $56x^2$
s. $-7u^2$ t. $-16c^2$ u. 0 v. $42mn$ w. As is x. $-169k^2$
y. 0 z. $6x$

2. a. $12x$ b. $35y$ c. $-6n$ d. $-20n$ e. 42 f. 64
g. z^2 h. $6n^2$ i. $-14m^2$ j. $-4p^2$ k. $-15a^2$ l. $21a$
m. -100 n. $80g$ o. $-48x$

3. $4n^2 + 7n = 4(\mathbf{2})^2 + 7(\mathbf{2}) = 4(4) + 7(2) = 16 + 14 = 30$,
whereas $11n^3 = 11(\mathbf{2})^3 = 11(8) = 88$
Therefore, $4n^2 + 7n \neq 11n^3$

4. a. $8x^2 - 7x + 9$ b. 0 c. $-4u^2 + 4u$ d. $-2a^2 - a$
e. $8x^2 - 6x$ f. $6y^2 - 4$ g. $-10x + 6$ h. $-6w^2 + 5w$

5. a. $5c^2 + 8c - 15$ b. $-7x^2 - 59x + 8$ c. $-4a^2 + 4a + 1$
d. $2w^2 - 10w + 10$ e. $-u^2 + 4u + 1$ f. 0
g. $-5x^2 - 4x - 13$ h. $-6n^2 - 14n + 4$

6. a. $9a^2 + 81a$ b. $30x^2 - 10x$ c. $-5y + 5y^2$
d. $5n^2 + n$ e. $25a^2 + 25a$ f. $21u^2 - 24u$
g. $-5m^2 - 5m$ h. $-30t^2 - 36t$ i. $-6x^2 + 9$

7. a. $12n$ b. $-24x$ c. $5b$ d. $-16y^2$ e. $6y$ f. $-6xz$

8. a. $28x$ b. $15a^2$ c. $6mn$ d. -49 e. $-39t$ f. $-49x^2$

9. a. $-6x$ b. $-24y^2 + 32y + 8$ c. $a^2 + 2a - 15$
d. 0 e. $-5g + 11$ f. $56x^2 + 40x$

"Formal education is but an incident in the lifetime of an individual. Most of us who have given the subject any study have come to realize that education is a continuous process ending only when ambition comes to a halt."

– R.I. Rees