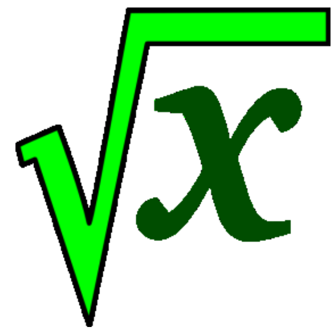
SOLVING QUADRATICS BY TAKING SQUARE ROOTS

❑ *INTRODUCTION*



Let's look at the quadratic equation

$$x^2 = 100$$

One way you may have learned to solve this equation is by **factoring**:

$$x^2 = 100 \quad \Rightarrow \quad x^2 - 100 = 0 \quad \Rightarrow \quad (x + 10)(x - 10) = 0$$

$$\Rightarrow \qquad x + 10 = 0 \ \text{ or } \ x - 10 = 0 \quad \Rightarrow \quad x = -10 \ \text{ or } \ x = 10$$

The solutions of the quadratic equation $x^2 = 100$ are simply $x = \pm\mathbf{10}$.

A second way you may know is to use the **Quadratic Formula**:

$x^2 = 100 \Rightarrow x^2 - 100 = 0$, which means that $a = 1$, $b = 0$, and $c = -100$. Placing these three values into the Quadratic Formula gives:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-0 \pm \sqrt{(0)^2 - 4(1)(-100)}}{2(1)} = \frac{\pm\sqrt{400}}{2} = \frac{\pm 20}{2} = \pm\,\mathbf{10}$$

But there's a third way (and easier, in this case) to solve a quadratic equation like $x^2 = 100$. Just take the square root of each side of the equation, <u>remembering that the number 100 has **TWO** square roots</u>, namely 10 and –10. Therefore, $x = \pm\mathbf{10}$, and neither factoring nor the Quadratic Formula is required for this simple quadratic equation.

For another example, let's solve the quadratic equation $n^2 = 30$. Remembering that 30 has <u>two</u> square roots, we calculate n to be $\pm\sqrt{\mathbf{30}}$.

For a third example, where we will need to simplify the radical, consider the quadratic equation $y^2 = 72$. When we take the square root of each side of the equation -- *and when we remember that* 72 *has* *<u>two</u>* *square roots* -- we see that

$$y^2 = 72 \quad \Rightarrow \quad y = \pm\sqrt{72} = \pm\sqrt{36 \cdot 2} = \pm\sqrt{36} \cdot \sqrt{2} = \pm 6\sqrt{2}$$

In short, the solutions of $y^2 = 72$ are $\mathbf{\pm 6\sqrt{2}}$.

❑ *The Square Root Theorem*

Now for the general statement:

> The solutions of the equation
> $x^2 = A$ are $x = \pm\sqrt{A}$

The Square Root Theorem

<u>Notes:</u>

1) The value of A in the Square Root Theorem is assumed to be zero or positive; that is, $A \geq 0$. Otherwise, the square root will not be a number that we use in Elementary Algebra.

2) How do students usually mess up this kind of equation? By forgetting to include <u>both</u> square roots (that is, they forget the "±" sign). DON'T MESS UP!

<u>EXAMPLE 1:</u> **Solve the quadratic equation: $(x+7)^2 = 81$**

<u>Solution:</u> According to the Square Root Theorem, we can remove the squaring by taking the square <u>root</u> of both sides of the equation, remembering that the number 81 has <u>two</u> square roots:

$(x+7)^2 = 81$ (the original equation)

$\Rightarrow \quad x+7 = \pm\sqrt{81}$ (the Square Root Theorem)

$\Rightarrow \quad x+7 = \pm 9$ ($\sqrt{81} = 9$)

$\Rightarrow \quad x = -7 \pm 9$ (subtract 7 from each side)

Using the plus sign yields $x = -7 + 9 = 2$.

Using the minus sign yields $x = -7 - 9 = -16$.

$$\boxed{x = 2 \text{ or } -16}$$

<u>EXAMPLE 2:</u> **Solve for y: $(y-3)^2 = 32$**

<u>Solution:</u> First we need to remove, or undo, the squaring in this quadratic equation. This is where we apply the Square Root Theorem:

$y - 3 = \pm\sqrt{32}$ (32 has <u>two</u> square roots)

To isolate the y, add 3 to both sides:

$y = 3 \pm \sqrt{32}$

Since $\sqrt{32} = \sqrt{16 \times 2} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$, we simplify our solution:

$$\boxed{y = 3 \pm 4\sqrt{2}}$$

Be sure it's clear to you that we have written <u>two</u> solutions to our quadratic equation:

$3 + 4\sqrt{2}$ and $3 - 4\sqrt{2}$

<u>EXAMPLE 3:</u> **Solve for n: $(n - 3)^2 = -49$**

<u>Solution:</u> Applying the Square Root Theorem to remove the squaring gives us the equation

$$n - 3 = \pm\sqrt{-49}$$

We needn't go any further; after all, the square root of a negative number doesn't exist in this class. So we're done right here, and we conclude that the equation has

No Solution

Homework

1. Solve each equation by applying the Square Root Theorem:

a. $x^2 = 144$	b. $y^2 = 51$	c. $z^2 = 72$
d. $a^2 = 0$	e. $b^2 = -9$	f. $(x + 1)^2 = 25$
g. $(n - 3)^2 = 100$	h. $(u + 10)^2 = 1$	i. $(a - 5)^2 = 32$
j. $(b + 7)^2 = 50$	k. $(w + 13)^2 = -4$	l. $(m - 3)^2 = 75$

❑ *SPLITTING RADICALS IN DIVISION*

Do you remember the rule about splitting the square root of a product: $\sqrt{ab} = \sqrt{a}\sqrt{b}$? The same kind of rule works for division. For example, we know that $\sqrt{\frac{9}{25}} = \frac{3}{5}$, since $\left(\frac{3}{5}\right)^2 = \frac{9}{25}$. Now let's work it out by "splitting" the radical:

$$\sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}\text{, the same answer!}$$

In short, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ [assuming that $a \geq 0$ and $b > 0$]

Homework

2. Calculate each square root by "splitting" the radical:

a. $\sqrt{\frac{1}{25}}$ b. $\sqrt{\frac{9}{49}}$ c. $\sqrt{\frac{16}{81}}$ d. $\sqrt{\frac{100}{121}}$ e. $\sqrt{\frac{144}{36}}$

We conclude this chapter with an example which requires us to "split" the square root of a fraction.

<u>EXAMPLE 4:</u> **Solve for z:** $\left(z - \frac{4}{5}\right)^2 = \frac{8}{25}$

<u>Solution:</u> Using the Square Root Theorem, we take the square root of each side of the equation, remembering that $\frac{8}{25}$ has <u>two</u> square roots:

$$z - \frac{4}{5} = \pm\sqrt{\frac{8}{25}} \quad \text{(the Square Root Theorem)}$$

$$\Rightarrow \quad z = \frac{4}{5} \pm \sqrt{\frac{8}{25}} \quad \text{(isolate the } z\text{)}$$

$$\Rightarrow \quad z = \frac{4}{5} \pm \frac{\sqrt{8}}{\sqrt{25}} \quad \text{(split the radical)}$$

$$\Rightarrow \quad z = \frac{4}{5} \pm \frac{2\sqrt{2}}{5} \quad \text{(simplify both square roots)}$$

$$\Rightarrow \quad \boxed{z = \frac{4 \pm 2\sqrt{2}}{5}} \quad \text{(combine into a single fraction)}$$

Again, note that we have found <u>two</u> solutions. They may be ugly, but both of them satisfy the equation.

Homework

3. Solve each equation by applying the Square Root Theorem:

a. $\left(x - \frac{1}{2}\right)^2 = \frac{3}{4}$ b. $\left(t + \frac{2}{3}\right)^2 = \frac{1}{9}$ c. $\left(z - \frac{4}{5}\right)^2 = \frac{19}{25}$

d. $\left(x + \frac{3}{5}\right)^2 = \frac{12}{25}$ e. $\left(b - \frac{9}{10}\right)^2 = \frac{81}{100}$ f. $\left(g - \frac{3}{7}\right)^2 = \frac{24}{49}$

Review Problems

4. Solve each equation by applying The Square Root Theorem:

a. $x^2 = 121$ b. $y^2 = 50$ c. $z^2 = 0$

d. $n^2 = -25$ e. $t^2 = 288$ f. $a^2 - 14 = 0$

g. $(x + 1)^2 = 75$ h. $(c - 3)^2 = 10$ i. $(x + 10)^2 = -1$

j. $\left(w + \frac{1}{2}\right)^2 = \frac{3}{4}$ k. $\left(u - \frac{4}{3}\right)^2 = \frac{26}{9}$ l. $\left(a + \frac{8}{5}\right)^2 = \frac{32}{25}$

Solutions

1. a. $x = \pm 12$ b. $y = \pm\sqrt{51}$ c. $z = \pm 6\sqrt{2}$

d. $a = 0$ e. No solution f. $x = 4, -6$

g. $n = 13, -7$ h. $u = -9, -11$ i. $a = 5 \pm 4\sqrt{2}$

j. $b = -7 \pm 5\sqrt{2}$ k. No solution l. $m = 3 \pm 5\sqrt{3}$

2. a. $\sqrt{\frac{1}{25}} = \frac{\sqrt{1}}{\sqrt{25}} = \frac{1}{5}$ b. $\frac{3}{7}$ c. $\frac{4}{9}$ d. $\frac{10}{11}$ e. 2

3. a. $\left(x - \frac{1}{2}\right)^2 = \frac{3}{4} \Rightarrow x - \frac{1}{2} = \pm\sqrt{\frac{3}{4}} \Rightarrow x = \frac{1}{2} \pm \frac{\sqrt{3}}{\sqrt{4}} = \frac{1}{2} \pm \frac{\sqrt{3}}{2} = \frac{1 \pm \sqrt{3}}{2}$

b. $t = -\frac{1}{3}, -1$ c. $z = \frac{4 \pm \sqrt{19}}{5}$ d. $x = \frac{-3 \pm 2\sqrt{3}}{5}$

e. $b = \frac{9}{5}, 0$ f. $g = \frac{3 \pm 2\sqrt{6}}{7}$

4. a. $x = \pm 11$ b. $y = \pm 5\sqrt{2}$ c. $z = 0$

d. No solution e. $t = \pm 12\sqrt{2}$ f. $a = \pm\sqrt{14}$

g. $x = -1 \pm 5\sqrt{3}$ h. $c = 3 \pm \sqrt{10}$ i. No solution

j. $w = \frac{-1 \pm \sqrt{3}}{2}$ k. $u = \frac{4 \pm \sqrt{26}}{3}$ l. $a = \frac{-8 \pm 4\sqrt{2}}{5}$

"Learning is a treasure

that will follow its owner everywhere."

Chinese Proverb