SCIENTIFIC NOTATION

❑ *INTRODUCTION*

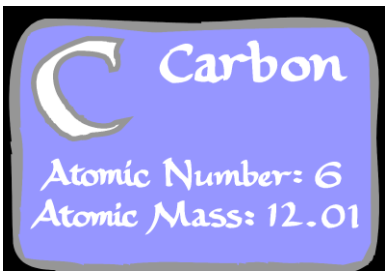
You might read that the distance from the Earth to the Sun is approximately 9.3×10^7 miles. This way of writing a number is called ***scientific notation***, and is used to write very large and very small numbers in a compact notation that's easier to understand than long strings of digits. To convert 9.3×10^7 into a regular number, we need only our knowledge of exponents:

$$\begin{aligned} & 9.3 \times 10^7 \\ = \; & 9.3 \times (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \\ = \; & 9.3 \times 10{,}000{,}000 \\ = \; & 93{,}000{,}000 \end{aligned}$$

Thus, 9.3×10^7 miles is simply 93 million miles. Why not just write "93 million miles," or 93,000,000 miles? Because with other numbers it might become quite cumbersome. If we were to expand, for instance, the number 6.02×10^{23}, we'd get "602" followed by 21 zeros:

$$6.02 \times 10^{23} = 602{,}000{,}000{,}000{,}000{,}000{,}000{,}000$$

Not only would it be horrendous to write it this way, most people wouldn't even know the name for a number that big -- thus the need for scientific notation.



6.02×10^{23} is the number of **atoms** in 12 grams of Carbon-12.

❑ MULTIPLYING AND DIVIDING BY POWERS OF 10

Before we begin the actual conversions, let's review how we multiply and divide by powers of 10.

7.06×10^6 is calculated by simply moving the decimal point 6 places to the right $\Rightarrow$ **7,060,000**

To multiply by a negative power of 10, move the decimal point to the left. For example, 6.09×10^{-5} = **0.0000609**

Can you see how we know whether to move the decimal point to the right or to left?

❑ THE DEFINITION OF SCIENTIFIC NOTATION

A number is in ***scientific notation*** if it's written in the form

$$M \times 10^C$$

where M is a number between 1 and 9.999… and C is an integer (whole numbers and their negatives).

Also notice that the M is being multiplied by the power of **10**. In the Introduction, we stated that the distance from the Earth to the Sun was 9.3×10^7 miles, a number written in *scientific notation*.

$$M \times 10^C$$

M is any number between 1 and 10 (excluding the 10), and C must come from the set $\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$.

The following are in scientific notation, and the reason is that in each case the operation is multiplication, the front number is between 1 and 10 (but not 10), and the exponent is an integer:

4.56×10^7 $\quad$ 9.7023×10^{-9} $\quad$ 8×10^0 $\quad$ 1×10^{34}

The following, however, are <u>not</u> in scientific notation:

23.9×10^{17}	The front number is too big.
0.72×10^{-67}	The front number is too small.
10×10^{44}	The front number is too big.
7.023×2^{34}	The base (the 2) is not 10.
$3.4 \times 10^{1/2}$	The exponent is not an integer.
$17.13 + 10^{19}$	It's an addition problem, not multiplication.

❑ *CONVERTING WITH SCIENTIFIC NOTATION*

<u>EXAMPLE 1:</u> **Convert each scientific notation number into a regular number (sometimes called standard notation):**

A. **2.73×10^{4}**

Move the decimal point 4 places to the right ⇒ **27,300**

B. **8.9×10^{1}**

Move the point 1 place to the right ⇒ **89**

C. **1.33×10^{0}**

Move the point 0 places to the right ⇒ **1.33**

[Or, since $10^{0} = 1$, the answer must be 1.33.]

D. **4.1×10^{-3}**

Move the point 3 places to the left ⇒ **0.0041**

E. **7.827×10^{-8}**

Move the point 8 places to the left ⇒ **0.00000007827**

EXAMPLE 2: **Convert each regular number into scientific notation:**

A. **540.92**

We need the decimal part of the scientific notation number to be between 1 and 10, so we move the decimal point 2 places to the left, giving 5.4092, which is equivalent to dividing by 100. We compensate this dividing by multiplying the new decimal by 100, or 10^2. We end up with $\mathbf{5.4092 \times 10^2}$.

B. **0.000056**

To get the decimal in the right spot for scientific notation, we need to move it 5 places to the right, which is essentially multiplying the number by 100,000; this produces 5.6. We balance this maneuver by "dividing" by 100,000, which is equivalent by multiplying by 10^{-5}. Thus, the given decimal is equal to $\mathbf{5.6 \times 10^{-5}}$.

Homework

1. Convert each scientific notation number into a regular number:

   a. 8.73×10^2 b. 2.09×10^5 c. 8.0×10^1 d. 2.36×10^{10}

   e. 5.99×10^{-2} f. 6.01×10^{-4} g. 1.0×10^{-6} h. 8.35×10^0

2. Convert each regular number into a scientific notation number:

   a. 9,200,000 b. 23,400,000,000 c. 34.56

   d. 0.0000123 e. 76,996.7 f. 0.0000000077

❑ CALCULATING WITH SCIENTIFIC NOTATION

EXAMPLE 3: **Calculate the product: $(2.3 \times 10^4)(1.7 \times 10^7)$**

Solution: Notice that the problem contains nothing but multiplications; we can therefore rearrange the four factors in any manner we please (associative and commutative properties).

$$
\begin{aligned}
& (2.3 \times 10^4)(1.7 \times 10^7) \\
= \quad & 2.3 \times 1.7 \times 10^4 \times 10^7 && \text{(rearrangement)} \\
= \quad & (2.3 \times 1.7) \times (10^4 \times 10^7) && \text{(regroup)} \\
= \quad & \mathbf{3.91 \times 10^{11}} && \text{(add the exponents)}
\end{aligned}
$$

EXAMPLE 4: **Find the quotient: $\dfrac{3.91 \times 10^{23}}{2.3 \times 10^{15}}$**

Solution: This one's a little tricky. Remember how we multiply fractions: $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$. Reading this rule the other way, we can take something like $\frac{xw}{yz}$ and split it into $\frac{x}{y} \times \frac{w}{z}$. Now for the problem.

$$\frac{3.91 \times 10^{23}}{2.3 \times 10^{15}} = \frac{3.91}{2.3} \times \frac{10^{23}}{10^{15}} = \mathbf{1.7 \times 10^8}$$

Homework

3. Calculate each product:

a. $(1.1 \times 10^{6})\ (2.42 \times 10^{4})$ b. $(2.0 \times 10^{5})\ (3.7 \times 10^{-8})$

c. $(4.1 \times 10^{6})\ (2.1 \times 10^{-4})$ d. $(1.9 \times 10^{-23})\ (2.3 \times 10^{-7})$

4. Calculate each quotient:

a. $\frac{4.42 \times 10^{15}}{1.7 \times 10^{13}}$ b. $\frac{8.0 \times 10^{9}}{2.0 \times 10^{20}}$

c. $\frac{9.2 \times 10^{-9}}{4.6 \times 10^{4}}$ d. $\frac{4.095 \times 10^{15}}{1.5 \times 10^{-8}}$

Solutions

1. a. 873 b. 209,000 c. 80 d. 23,600,000,000

e. 0.0599 f. 0.000601 g. 0.000001 h. 8.35

2. a. 9.2×10^{6} b. 2.34×10^{10} c. 3.456×10^{1}

d. 1.23×10^{-5} e. 7.69967×10^{4} f. 7.7×10^{-9}

3. a. 2.662×10^{10} b. 7.4×10^{-3}

c. 8.61×10^{2} d. 4.37×10^{-30}

4. a. 2.6×10^{2} b. 4×10^{-11}

c. 2×10^{-13} d. 2.73×10^{23}

What sculpture is to a block of marble, education is to the human soul.”

Joseph Addison