SERIES

A mathematical ***series*** is a fancy term for **sum**, or summation. A **finite** series might be the summation

$$1 + 2 + 5 + 6 + 10 \qquad \text{(which has a sum of 24)}$$

Sometimes, if there's a nice pattern, a series can be written in "**sigma**" notation. The Greek capital letter sigma, Σ, is used to represent the summation (since sigma and sum both begin with the letter *s*). The following examples should explain how this notation is to be read and calculated.

EXAMPLE 1: **Evaluate:** $\displaystyle\sum_{k=2}^{5}(2k+1)$

Solution: The expression which determines the numbers we will add together is $2k + 1$. Below the sigma sign is the starting value of k, in this case 2; above the sigma sign is the ending value of k, in this case 5. So k starts at 2 and ends at 5, but what does k do in between? We agree that it goes up by one—in other words, k will go 2, 3, 4, and then 5. Check it out, remembering that the sigma sign, Σ, means ADD:

$$\begin{aligned}\sum_{k=2}^{5}(2k+1) &= \underset{(k=2)}{(2\cdot\mathbf{2}+1)} + \underset{(k=3)}{(2\cdot\mathbf{3}+1)} + \underset{(k=4)}{(2\cdot\mathbf{4}+1)} + \underset{(k=5)}{(2\cdot\mathbf{5}+1)} \\ &= 5 + 7 + 9 + 11 \\ &= \boxed{\mathbf{32}}\end{aligned}$$

EXAMPLE 2: **Evaluate:** $\sum_{n=0}^{3}(n^2 - n)$

Solution: Does it matter that Example 10 used the variable k and this example uses the variable n? Not at all—the variable used makes no difference in the final answer; it's just a placeholder. The values of n will be 0, 1, 2, and 3. For each value of n we evaluate the expression $n^2 - n$. Then we add up the results.

$(n = 0)$ $(n = 1)$ $(n = 2)$ $(n = 3)$

$$\sum_{n=0}^{3}(n^2 - n) = (0^2 - 0) + (1^2 - 1) + (2^2 - 2) + (3^2 - 3)$$

$$= 0 + 0 + 2 + 6$$

$$= \boxed{8}$$

Homework

Evaluate each series (calculate each sum):

1. $\sum_{k=3}^{5}(7k - 1)$
2. $\sum_{n=2}^{5}(n^2 + n)$
3. $\sum_{k=1}^{6}\frac{1}{2^k}$
4. $\sum_{j=0}^{4}3^j$
5. $\sum_{n=16}^{16}\frac{1}{2}\sqrt{n}$
6. $\sum_{k=3}^{5}\frac{1}{k}$
7. $\sum_{m=0}^{2}\frac{1}{m+1}$
8. $\sum_{t=-1}^{4}2t$
9. $\sum_{j=-2}^{2}j^3$

Review Problems

10. $\sum_{n=2}^{5}(n^2+n) =$

11. $\sum_{k=1}^{6}\frac{1}{2^k} =$

12. $\sum_{j=0}^{4}3^j =$

13. $\sum_{n=16}^{16}\frac{1}{2}\sqrt{n} =$

14. $\sum_{k=1}^{10}k =$

15. $\sum_{n=0}^{4}2^{n-2} =$

16. True/False:

a. $\sum_{i=2}^{5}i^2 = 54$

b. $\sum_{j=0}^{3}2^j = 16$

c. $\sum_{k=-1}^{1}k^3 = 3$

d. $\sum_{L=0}^{4}\sqrt{L} = 3+\sqrt{2}+\sqrt{3}$

Solutions

1. 81 **2.** 68 **3.** $\frac{63}{64}$ **4.** 121

5. 2 **6.** $\frac{47}{60}$ **7.** $\frac{11}{6}$ **8.** 18

9. 0 **10.** 68 **11.** $\frac{63}{64}$ **12.** 121

13. 2 **14.** 55 **15.** $\frac{31}{4}$

16. a. T b. F c. F d. T

❑ TO ∞ AND BEYOND

A. Evaluate: $\sum_{k=1}^{99}\left(\frac{1}{k}-\frac{1}{k+1}\right)$

B. Evaluate: $\sum_{k=1}^{\infty}\frac{1}{2^k}$

"The highest activity a human being can attain is learning for understanding, because to understand is to be free."

– Baruch Spinoza