# ADDING AND SUBTRACTING SIGNED NUMBERS

## ❑ *LET'S PLAY JEOPARDY!*


Each answer in Jeopardy is worth a specific number of dollars. Your score goes up or down depending on whether you get the question right or wrong. It is also possible for scores to go below zero -- into the negative numbers.

In our first example, you should use your intuition, if possible. But until you're comfortable with that method, you might try picturing a number line: A correct response (that is, adding) moves you to the right on the number line, while an incorrect response (that is, subtracting) moves you to the left on the number line.

EXAMPLE 1:

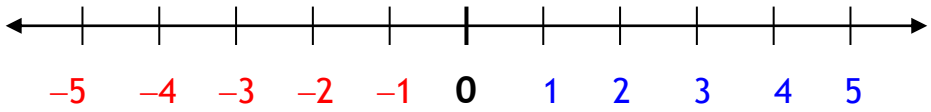
| $ Current Amount $ | $ Value of Question $ | Right or Wrong | Calculation | $ New Amount $ |
|---|---|---|---|---|
| 20 | 10 | right | 20 + 10 | 30 |
| 15 | 5 | wrong | 15 – 5 | 10 |
| 12 | 15 | wrong | 12 – 15 | –3 |
| –10 | 5 | right | –10 + 5 | –5 |
| –20 | 30 | right | –20 + 30 | 10 |
| –8 | 5 | wrong | –8 – 5 | –13 |
| 25 | 25 | wrong | 25 – 25 | 0 |
| –200 | 200 | right | –200 + 200 | 0 |
| 0 | 400 | right | 0 + 400 | 400 |
| 0 | 500 | wrong | 0 – 500 | –500 |

# Homework

1. Fill in the blanks:

| $ Current Amount $ | $ Value of Question $ | Right or Wrong | Calculation | $ New Amount $ |
|---|---|---|---|---|
| 25 | 10 | right | | |
| 12 | 5 | wrong | | |
| 10 | 15 | wrong | | |
| –15 | 5 | right | | |
| –25 | 30 | right | | |
| –8 | 7 | wrong | | |
| 29 | 29 | wrong | | |
| –300 | 300 | right | | |
| 0 | 800 | right | | |
| 0 | 300 | wrong | | |

## ❑ *THE NUMBER LINE APPROACH TO ADDING AND SUBTRACTING SIGNED NUMBERS*




EXAMPLE 2:

A. $3 + 2 = 5$

Start at 3 on the number line; move 2 units to the right, and you end up at 5.

B. $-4 + 5 = 1$

Start at –4; move 5 units to the right, and you end up at 1.

C. $-5 + 3 = -2$

Start at –5; move 3 units to the right, and you end up at –2.

D. $5 - 3 = 2$

Start at 5; move 3 units to the left, and you end up at 2.

E. $3 - 7 = -4$

Start at 3; move 7 units to the left, and you end up at –4.

F. $-3 - 2 = -5$

Start at –3; move 2 units to the left, and you end up at –5.

All of these results could have been obtained by thinking about Jeopardy. You may pick any method you like to add and subtract positive numbers, including anything you learned in a previous course; just be sure you can do it really well. Now for some problems where we add and subtract negative numbers.

EXAMPLE 3:

A. $5 + (-2) = 3$

Since we know that adding a positive number moves you to the right on the number line, it's reasonable to suppose that adding a negative number moves you to the left on the number line.

Start at 5 on the number line; move 2 units to the left, and you end up at 3.

B. $3 + (-6) = -3$

Start at 3; move 6 units to the left, and you end up at –3.

C. $-1 + (-4) = -5$

Start at –1; move 4 units to the left, and you wind up at –5.

Now for the strangest example: 3 – (–2). Here's the logic: Since we've learned that subtracting a positive number moves you to the left on the number line, we figure that subtracting a negative number moves you to the right on the number line.

D. $3 - (-2) = 5$

Start at 3; move 2 units to the right, and you end up at 5.

E. $-7 - (-5) = -2$

Start at –7; move 5 units to the right, and you end up at –2.

Let's look at the four conclusions we've reached regarding the adding and subtracting of positive and negative numbers. Assuming $b$ is a positive number:

| | | |
|---|---|---|
| (i) | $a + b$ | Move $b$ units to the right |
| (ii) | $a - b$ | Move $b$ units to the left |
| (iii) | $a + (-b)$ | Move $b$ units to the left |
| (iv) | $a - (-b)$ | Move $b$ units to the right |

## Homework

2. Perform each addition or subtraction problem:

a. 17 + 3 b. 16 – 5 c. 7 – 8 d. 12 – 12

e. 23 – 30 f. –2 + 10 g. –3 + 2 h. –12 – 3

| | | | |
|---|---|---|---|
| i. $8 - 100$ | j. $-30 + 40$ | k. $-20 + 5$ | l. $-80 - 20$ |
| m. $0 + 32$ | n. $0 - 32$ | o. $88 + 0$ | p. $34 - 0$ |
| q. $-10 - 15$ | r. $-23 + 80$ | s. $-7 + 2$ | t. $-18 - 18$ |
| u. $9 - 12$ | v. $-5 + 9$ | w. $-10 - 3$ | x. $-10 + 2$ |
| y. $-30 - 4$ | z. $3 - 100$ | | |

3. Evaluate (simplify) each expression:

| | | | |
|---|---|---|---|
| a. $17 + 3$ | b. $-8 + 3$ | c. $-8 + (-1)$ | d. $7 + (-3)$ |
| e. $(-3) + (-10)$ | f. $5 + (-13)$ | g. $-9 + 0$ | h. $-7 + 12$ |
| i. $-20 + 10$ | j. $-30 + (-2)$ | k. $0 + (-3)$ | l. $-10 + 10$ |
| m. $-12 + 7$ | n. $-3 + (-20)$ | o. $9 + (-9)$ | p. $1 + (-12)$ |

4. Evaluate (simplify) each expression:

| | | | |
|---|---|---|---|
| a. $7 - 5$ | b. $9 - 10$ | c. $6 - 6$ | d. $12 - 25$ |
| e. $-5 - 4$ | f. $-10 - 10$ | g. $13 - 20$ | h. $-1 - 99$ |
| i. $12 - (-3)$ | j. $-1 - (-4)$ | k. $-14 - (-3)$ | l. $-2 - (-2)$ |
| m. $-3 - 17$ | n. $20 - (-21)$ | o. $-5 - (-13)$ | p. $-9 - (-7)$ |

NOTE: You may have already learned that addition and multiplication are **commutative** operations, since $a + b = b + a$ and $ab = ba$ for any values of $a$ and $b$. Is subtraction commutative? That is, does $a - b = b - a$ for all choices of $a$ and $b$? Of course not. After all, $7 - 2 = 5$, while $2 - 7 = -5$. Thus, "commuting" the 7 and the 2 around the subtraction sign results in different answers. Thus, subtraction is not a commutative operation (nor is division).

EXAMPLE 4:

A. $7 + (-3) + 8$

$= 4 + 8$ (start at 7; move 3 to the left)

$= \mathbf{12}$ (now move 8 to the right)

B. $-13 - 3 + (-5)$

$= -16 + (-5)$ (start at –13; move 3 to the left)

$= \mathbf{-21}$ (move 5 to the left)

C. $-8 - (-1) - (-3)$

$= -7 - (-3)$ (start at –8; move 1 to the right

$= \mathbf{-4}$ (move 3 to the right)

D. $13 + (-7) - (-1)$

$= 6 - (-1)$ (start at 13; move 7 to the left)

$= \mathbf{7}$ (move 1 to the right)

E. $-12 + (-3) - 9 - (-2)$

Start at –12; move 3 to the left:

$= -15 - 9 - (-2)$

Now you're at –15; move 9 to the left:

$= -24 - (-2)$

Now you're at –24; move 2 to the right:

$= \mathbf{-22}$ and we're finally done!

# Homework

5. Evaluate (simplify) each expression:

a. $-7 - (-3) - (-12)$

b. $8 - (-2) - (-9)$

c. $5 + (-11) + 1 + (-8)$

d. $-3 - (-2) - (-11) - 7 - 3$

e. $-4 - (-7) + 12$

f. $11 + 6 + (-6) + 6$

g. $-6 + 9 - 12 + 11$

h. $5 - (-6) + (-4) - 7 - (-3)$

i. $-2 + 2 + 9 - (-6) + 8$

j. $-6 - 6 - 1$

k. $6 - 4 - 7 - (-10)$

l. $2 - (-12) + (-7)$

m. $6 + 2 - 8 + 4 + 7$

n. $-9 - (-5) + 10 + (-11)$

o. $-9 + 9 - 5 - 12$

p. $-2 - 11 - 3$

# Solutions

**1**.

| $ Current Amount $ | $ Value of Question $ | Right or Wrong | Calculation | $ New Amount $ |
|---|---|---|---|---|
| 25 | 10 | right | **25 + 10** | **35** |
| 12 | 5 | wrong | **12 – 5** | **7** |
| 10 | 15 | wrong | **10 – 15** | **–5** |
| –15 | 5 | right | **–15 + 5** | **–10** |
| –25 | 30 | right | **–25 + 30** | **5** |
| –8 | 7 | wrong | **–8 – 7** | **–15** |
| 29 | 29 | wrong | **29 – 29** | **0** |

| $ Current Amount $ | $ Value of Question $ | Right or Wrong | Calculation | $ New Amount $ |
|---|---|---|---|---|
| –300 | 300 | right | **–300 + 300** | **0** |
| 0 | 800 | right | **0 + 800** | **800** |
| 0 | 300 | wrong | **0 – 300** | **–300** |

**2**. a. 20 b. 11 c. –1 d. 0 e. –7 f. 8 g. –1 h. –15 i. –92
j. 10 k. –15 l. –100 m. 32 n. –32 o. 88 p. 34 q. –25 r. 57
s. –5 t. –36 u. –3 v. 4 w. –13 x. –8 y. –34 z. –97

**3**. a. 20 b. –5 c. –9 d. 4 e. –13 f. –8 g. –9 h. 5 i. –10
j. –32 k. –3 l. 0 m. –5 n. –23 o. 0 p. –11

**4**. a. 2 b. –1 c. 0 d. –13 e. –9 f. –20 g. –7 h. –100
i. 15 j. 3 k. –11 l. 0 m. –20 n. 41 o. 8 p. –2

**5**. a. 8 b. 19 c. –13 d. 0 e. 15 f. 17 g. 2 h. 3
i. 23 j. – 13 k. 5 l. 7 m. 11 n. –5 o. –17 p. –16

**"The important thing is to not stop questioning."**

– Albert Einstein