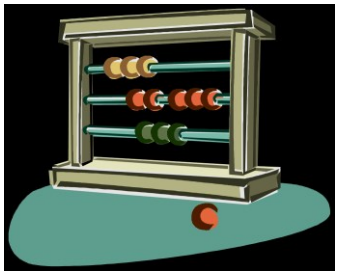
MULTIPLYING AND DIVIDING SIGNED NUMBERS

❑ *INTRODUCTION*



You may have learned how to solve an equation like $7n = 35$: You divide each side of the equation by 7, and conclude that $n = 5$. What if we come across an equation like

$$2x = -14\ ?$$

Our instinct should be to divide each side of the equation by 2 in order to isolate the x. But now we're faced with dividing a negative number (the -14) by a positive number (the 2). Not to worry -- we'll figure it out.

❑ *MULTIPLYING SIGNED NUMBERS*

Recall that the result of multiplying two numbers is called the **product** of the two numbers. So, for instance, the product of 3 and 5 is 15. Some ways to represent the product of 3 and 5 are

$3 \cdot 5$ 3×5 $3(5)$ $(3)5$ $(3)(5)$ All these products equal 15

Instead of using parentheses to indicate multiplication, square brackets are sometimes used. So $[3][5] = 15$ also.

Recall the ***commutative property of multiplication***, where the order in which you multiply a pair of numbers makes no difference in the product: $ab = ba$ for any numbers a and b.

Also recall the ***associative property of multiplication***, where parentheses can be "shifted" without changing the answer:

$(ab)c = a(bc)$ for any numbers a, b, and c

Positive Times Positive

What should 6×4 be? Luckily, the answer is exactly what you would expect. In other words, what we learned as kids still holds: **A positive number times a positive number is positive**.

Positive Times Negative

This one's not so obvious. Consider the multiplication problem (7)(–1). We'll determine the answer by sneaking up on the problem while it's not looking:

(7)(2) = 14	since a positive times a positive is positive
(7)(1) = 7	same rule – or – anything times 1 is itself
(7)(0) = 0	anything times 0 is 0
(7)(–1) = ???	what should the product be?

What comes next in the sequence 14, 7, 0, ? Since this sequence of numbers is <u>decreasing</u> by 7 at each step, the next number in the sequence must be –7, and we see that (7)(–1) = –7. It appears that **a positive number times a negative number is negative**.

Negative Times Positive

To calculate (–3)(4), first reverse the order of the factors (commutative property): (–3)(4) = (4)(–3), which is now a product of a positive with a negative. By the previous rule, we know the answer is –12. Thus, **a negative number times a positive number is negative**.

Negative Times Negative

Now for the most interesting situation, the product of two negative numbers -- for example (–5)(–1). We'll get a running start and see what emerges.

$(-5)(2) = -10$		since a negative times a positive is negative
$(-5)(1) = -5$		same rule – or – anything times 1 is itself
$(-5)(0) = 0$		anything times 0 is 0
$(-5)(-1) =$???		what should the product be?

What comes next in the sequence of answers –10, –5, 0 ? Since this sequence of numbers is <u>increasing</u> by 5 at each step, it follows that the next number is 5, giving us the result:

$(-5)(-1) = 5$

We've reached the inescapable conclusion that **a negative number times a negative number is <u>positive</u>**!

See if you can deduce the two rules in the following box from the four rules stated above.

Multiplying Signed Numbers:

If the signs are the same, the product is positive.

If the signs are different, the product is negative.

Homework

1. Find the product:

a. (17)(3)	b. (–4)(7)	c. 3(–10)	d. (–3)(–4)
e. –7(–2)	f. 2(–7)	g. –1(8)	h. –1(–9)
i. (1)(–134)	j. (–765)(0)	k. -3×-4	l. $7 \cdot -10$
m. (–18)(–2)	n. 7(–3)	o. –8(7)	p. $3 \cdot -9$
q. (–2)(99)	r. (–1)(–7)	s. (7)(–10)	t. (–4)(–5)

2. Find the product:

a. (0.2)(0.3) b. (–0.1)(0.1) c. (2.1)(–3) d. (–0.3)(–0.4)
e. (–0.3)(5) f. 2(–1.77) g. (–0.1)(–0.2) h. 17(–0.2)

3. Find the product in reduced form:

a. $\frac{2}{3}\times\frac{1}{5}$ b. $-\frac{2}{3}\cdot\frac{3}{4}$ c. $\frac{4}{5}\left(-\frac{5}{4}\right)$ d. $(-2)\left(-\frac{7}{2}\right)$
e. $\frac{4}{5}\times-\frac{10}{3}$ f. $\left[-\frac{1}{2}\right]\left[\frac{2}{99}\right]$ g. $\left(-\frac{2}{3}\right)\left(-\frac{3}{2}\right)$ h. $10\left(-\frac{3}{40}\right)$

4. Find the product:

Example: (–2)(3)(–4)
= (–6)(–4) (multiply the first two factors)
= 24

a. (–2)(–3)(4) b. 7 × –6 × 3 c. (–1)(–2)(–3)(–4)
d. 4 · 4 · 4(–1)(3) e. 5(–1)(–1)(–2) f. 4(–3)(2)(–8)
g. (–2)(–1)(–3) h. (–7)(6)(2)(–1) i. (–1)(–1)(–1)(8)
j. (–1)(–3)(–5)(–7) k. –3(–4)(5)2 l. (–2)(–2)(–1)(–3)(–5)0

❑ *DIVIDING SIGNED NUMBERS*

The secret to the division rules for signed numbers is the fact that division is checked by multiplication. For example,

$$\frac{56}{7} = 8, \text{ precisely because } 8 \times 7 = 56$$

Positive Divided by Positive

$\frac{6}{2} = 3$, since (3)(2) = 6.

Therefore, **a positive number divided by a positive number is positive**.

Negative Divided by Positive

$\frac{-6}{2} = -3$, because $(-3)(2) = -6$.

Thus**, a negative number divided by a positive number is negative.**

Positive Divided by Negative

$\frac{6}{-2} = -3$, which is checked by seeing that $(-3)(-2) = 6$.

Conclusion: **a positive number divided by a negative number is negative.**

Negative Divided by Negative

$\frac{-6}{-2} = 3$, which is confirmed by the fact that $(3)(-2) = -6$.

We see that **a negative number divided by a negative number is positive.**

NOTES: 1. Do these four rules for dividing signed numbers remind you of anything? The rules for <u>dividing</u> signed numbers are the <u>same</u> as the rules for <u>multiplying</u> signed numbers. (Now, that's nice.)

2. Note that $\frac{-10}{2} = \frac{10}{-2} = -\frac{10}{2}$,

because each of these three division problems has a quotient of –5. In general, the following three fractions are equal:

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$$

Homework

5. Using the logic above, which explains how signed numbers are divided, explain why each division problem is correct:

a. $\frac{22}{11} = 2$

b. $\frac{-21}{7} = -3$

c. $\frac{88}{-22} = -4$

d. $\frac{-100}{-20} = 5$

Here's the bottom line for dividing signed numbers:

Dividing Signed Numbers:

If the signs are the same, the quotient is positive.

If the signs are different, the quotient is negative.

EXAMPLE 1: **Find each quotient:**

A. $\frac{45}{-9} = -\mathbf{5}$ (positive divided by negative is negative)

B. $\frac{-250}{-25} = \mathbf{10}$ (negative divided by negative is positive)

C. $\frac{-21}{15} = -\frac{21}{15} = -\frac{\mathbf{7}}{\mathbf{5}}$ (negative divided by positive is negative)

D. $\frac{-8}{-12} = \frac{8}{12} = \frac{\mathbf{2}}{\mathbf{3}}$ (negative divided by negative is positive)

E. $\frac{18}{-7} = -\frac{\mathbf{18}}{\mathbf{7}}$ (positive divided by negative is negative)

F. $\frac{-4}{-9} = \frac{\mathbf{4}}{\mathbf{9}}$ (negative divided by negative is positive)

Homework

6. Find the quotient (leave answers as simplified fractions):

a. $\frac{30}{5}$ b. $\frac{60}{-10}$ c. $\frac{-100}{20}$ d. $\frac{-14}{-2}$

e. $\frac{3}{6}$ f. $\frac{-18}{4}$ g. $\frac{24}{-5}$ h. $\frac{-34}{-24}$

i. $\frac{-1}{9}$ j. $\frac{-345}{-345}$ k. $\frac{10}{-10}$ l. $\frac{-120}{-70}$

7. Evaluate each expression:

a. $\frac{5-1}{1-2}$ b. $\frac{10(-2)}{-4}$ c. $\frac{3^2-9}{7}$

d. $\frac{18-2}{2-18}$ e. $\frac{1+2+3}{-2-4}$ f. $\frac{3^2-1^3}{6-5}$

8. Evaluate each expression:

Example: $4 - 5[-3 + 2(3 - 6)]$

$= 4 - 5[-3 + 2(-3)]$ (start with innermost parentheses)

$= 4 - 5[-3 - 6]$ (multiply before adding)

$= 4 - 5[-9]$ (now for the brackets)

$= 4 + 45$ (multiply before subtracting)

$= \mathbf{49}$ (and we're done)

Evaluate each expression:

a. $2[5 - 9(-2)]$ b. $-3[1 + 2(-5)]$

c. $4 + 3[2 - 3(4)]$ d. $4 - 5[1 + 5(1 - 4)]$

e. $10 - 8[2(3) - 4(2)]$ f. $7 - 2[8 - 2(3 - 4)]$

g. $4 - 7[-5 - (2 - 9)]$ h. $10 - 2[10 - 2(3 - 7)]$

Review Problems

9. A negative divided by a negative is positive. Explain why.

10. a. $(-5)(-2)(-3) =$ b. $\frac{45}{-9} =$ c. $\frac{-80}{25} =$

d. $\frac{-12}{-48} =$ e. $\frac{2-8-10}{12-1-5} =$

11. a. $(-5)(2)(-4) =$ b. $\frac{-45}{-9} =$ c. $\frac{80}{-15} =$

d. $\frac{-36}{-48} =$ e. $\frac{2-8+10}{12+1-5} =$

12. $4 - 3[2(-3) - 5(8 - 6)] =$

The entire chapter can be summarized like this:

For both multiplying and dividing,

Same signs $\Rightarrow$ Positive answer

Different signs $\Rightarrow$ Negative answer

Solutions

1. a. 51 b. –28 c. –30 d. 12 e. 14 f. –14
g. –8 h. 9 i. –134 j. 0 k. 12 l. –70
m. 36 n. –21 o. –56 p. –27 q. –198 r. 7

s. -70 t. 20

2. a. 0.06 b. -0.01 c. -6.3 d. 0.12 e. -1.5
f. -3.54
g. 0.02 h. -3.4

3. a. $\frac{2}{15}$ b. $-\frac{1}{2}$ c. -1 d. 7 e. $-\frac{8}{3}$ f. $-\frac{1}{99}$
g. 1 h. $-\frac{3}{4}$

4. a. 24 b. -126 c. 24 d. -192 e. -10 f. 192
g. -6 h. 84 i. -8 j. 105 k. 120 l. 0

5. a. $(2)(11) = 22$ b. $(-3)(7) = -21$
c. $(-4)(-22) = 88$ d. $(5)(-20) = -100$

6. a. 6 b. -6 c. -5 d. 7 e. $\frac{1}{2}$
f. $-\frac{9}{2}$ g. $-\frac{24}{5}$ h. $\frac{17}{12}$ i. $-\frac{1}{9}$ j. 1
k. -1 l. $\frac{12}{7}$

7. a. -4 b. 5 c. 0 d. -1 e. -1 f. 8

8. a. 46 b. 27 c. -26 d. 74
e. 26 f. -13 g. -10 h. -26

9. $\frac{\text{neg}}{\text{neg}}$ = pos because when the division is checked by multiplication, we confirm that pos × neg = neg ✓

10. a. -30 b. -5 c. $-\frac{16}{5}$ d. $\frac{1}{4}$ e. $-\frac{8}{3}$

11. a. 40 b. 5 c. $-\frac{16}{3}$ d. $\frac{3}{4}$ e. $\frac{1}{2}$

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"Let us think of education as the means of developing our greatest abilities, because in each of us there is a private hope and dream which, fulfilled, can be translated into benefit for everyone and greater strength of the nation."

– John F. Kennedy