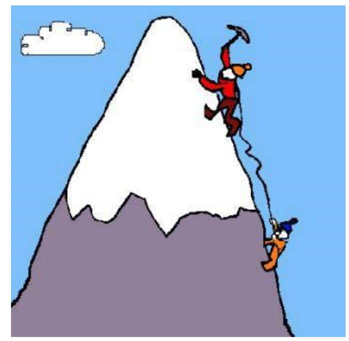
SLOPE: RISE/RUN

❑ *INTRODUCTION*



A line has many attributes, or characteristics. Two of the most important are its *intercepts* and its *slope*. The intercepts tell us where the line crosses the x-axis and the y-axis; they are very good reference points. The ***slope*** of a line tells us how *steep* the line is -- it's kind of like the angle that a line makes, and is a concept used in economics, chemistry, statistics, construction, and mountain climbing.

❑ *SLOPE*

A trucker is keenly aware of the *grade*, or angle, of the road on which a truck travels -- it determines the speed limit and the proper gear that the truck needs to be in. A roofer is concerned with the *pitch*, or steepness, of a roof. A construction worker needs to make sure that a wheelchair ramp has the correct *angle* with the street or sidewalk. All of these ideas are examples of the concept "*steepness*."

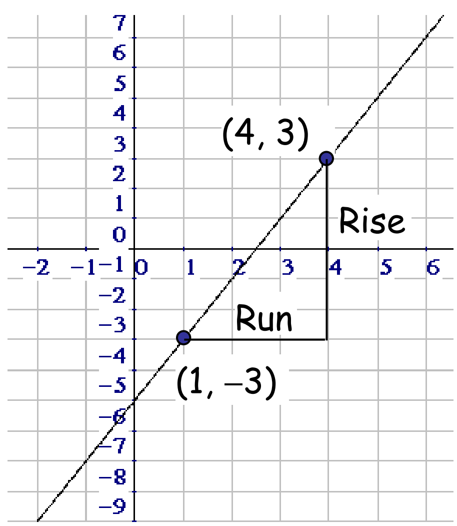
We'll use the term ***slope*** to represent steepness, and give it the letter m (I don't know why -- maybe m for mountain?). Our definition of slope in this course and all future math courses (and chemistry, economics, and nursing courses) is as follows:

$$m = \frac{\text{rise}}{\text{run}}$$

As we'll see shortly, a **rise** is a vertical (up/down) change, while a **run** is a horizontal (left/right) change. Slope is defined as the *ratio* of the rise to the run; we can also say that slope is the *quotient* of the rise and the run.

<u>EXAMPLE 1:</u> **Graph the line $y = 2x - 5$ and determine its slope.**

<u>Solution:</u> Let's calculate a couple of points by choosing some random x-values. If we let $x = 1$, then $y = -3$, so the point $(1, -3)$ is on the line. And if we let $x = 4$, then $y = 3$, giving us the point $(4, 3)$. We could calculate more points for our line, but let's cut to the chase and graph the line given the two points just computed.



Notice that we've constructed a right triangle using the line segment between the two given points as the hypotenuse. The rise and run are then just the lengths of the legs of the triangle. Counting squares from left to right along the bottom of the triangle, we see that the run is 3. Counting squares up the side of the triangle yields a rise of 6. Using the slope formula, we can calculate the slope of the line:

$$m = \frac{\text{rise}}{\text{run}} = \frac{6}{3} = \boxed{2}$$

Note: The concept of slope is dimensionless; that is, slope has no units. Here's why: Suppose that the units in the triangle are in feet. Then the slope is

$$m = \frac{\text{rise}}{\text{run}} = \frac{6\,\text{ft}}{3\,\text{ft}} = \frac{6\,\cancel{\text{ft}}}{3\,\cancel{\text{ft}}} = \mathbf{2} \text{ (since the feet cancel out)}$$

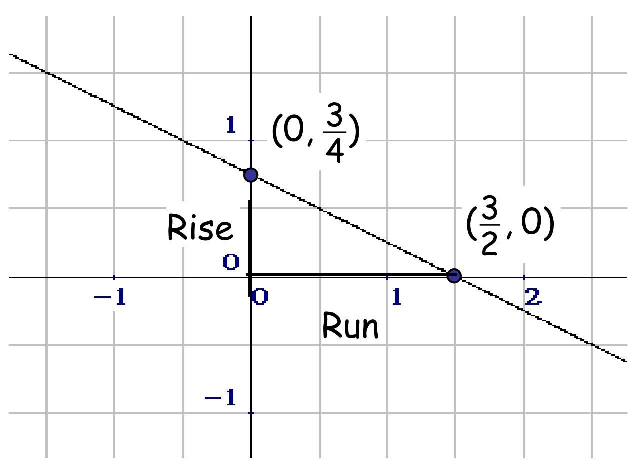
<u>EXAMPLE 2:</u> **Find the slope of the line $2x + 4y = 3$.**

<u>Solution:</u> To graph this line, let's calculate the two intercepts (since they're generally the easiest points to calculate). Set $x = 0$ to get

$$2(\mathbf{0}) + 4y = 3 \Rightarrow 4y = 3 \Rightarrow y = \frac{3}{4}$$

Thus, the y-intercept is $(0, \frac{3}{4})$. If we set $y = 0$, we can solve for x:

$2x + 4(\mathbf{0}) = 3 \Rightarrow 2x = 3 \Rightarrow x = \frac{3}{2}$, which implies that the x-intercept is $(\frac{3}{2}, 0)$. Plotting these two intercepts gives us our line:



As we move from left to right, from the y-intercept to the x-intercept, we notice that the rise is actually a drop -- this means that the rise is negative. Since the height of the triangle is $\frac{3}{4}$, we conclude that the "rise" is $-\frac{3}{4}$. Since the run is from left to right, the run is positive $\frac{3}{2}$. Now we're ready for the calculation:

$$m = \frac{\text{rise}}{\text{run}} = \frac{-\frac{3}{4}}{\frac{3}{2}} = -\frac{3}{4} \div \frac{3}{2} = -\frac{3}{4} \cdot \frac{2}{3} = \boxed{-\frac{1}{2}}$$

Note: Instead of moving from left to right, from the y-intercept to the x-intercept, we could also have moved from right to left, from the x-intercept to the y-intercept. In this case, the rise is positive because we're moving up, but the run is negative because we're moving to the left. This will still give us the same answer, since now the calculation looks like:

$$m = \frac{\text{rise}}{\text{run}} = \frac{\frac{3}{4}}{-\frac{3}{2}} = \frac{3}{4} \div -\frac{3}{2} = \frac{3}{4} \cdot -\frac{2}{3} = -\mathbf{\frac{1}{2}}$$

Homework

1. For each pair of points, plot them on a grid, find the rise and the run, and then use the formula for slope to calculate the *slope* of the line connecting the two points:

   a. (2, 3), (4, 7) b. (–3, 0), (0, 6) c. (1, –3), (–2, 5)

   d. (2, 2), (7, 7) e. (–3, –3), (0, 0) f. (–1, –2), (3, –5)

2. Find the *slope* of the given line by graphing the line and using the rise and run. You may, of course, use any two points on the line to calculate the rise and the run:

a. $y = x + 3$	b. $y = 2x - 1$	c. $y = -2x + 3$
d. $y = 3x + 1$	e. $y = -3x - 2$	f. $y = -x + 2$
g. $x + 2y = 4$	h. $2x - 3y = 1$	i. $3x - y = 3$
j. $-3x + 2y = 6$	k. $2x + 5y = 10$	l. $3x - 4y = -8$

Solutions

1.	a. 2	b. 2	c. $-\frac{8}{3}$	d. 1	e. 1	f. $-\frac{3}{4}$
2.	a. 1	b. 2	c. -2	d. 3	e. -3	f. -1
	g. $-\frac{1}{2}$	h. $\frac{2}{3}$	i. 3	j. $\frac{3}{2}$	k. $-\frac{2}{5}$	l. $\frac{3}{4}$

"Human history becomes more and more a race between education and catastrophe."

– H.G. Wells (1866-1946)