SYSTEMS OF EQUATIONS: THE ELIMINATION METHOD

❑ *THE ELIMINATION METHOD*

To solve the applications in future chapters, we need to be able to solve two equations in two variables. A method that works very well in many cases is called the ***Elimination Method***. We multiply one or both equations by appropriate numbers (whatever that means), **add** the resulting equations to eliminate a variable, and then solve for the variable that survived.

EXAMPLE 1: **Solve the system:** $\mathbf{5x - 2y = 20}$
$\mathbf{3x + 7y = -29}$

Solution: In the Elimination Method we may eliminate either variable. But there's a certain "orderliness" that comes in handy in future math courses if we always eliminate the first variable, so in this case we will eliminate the *x*. As mentioned above, we multiply one or both equations by some numbers, add the resulting equations to kill off one of the variables, and then solve for the variable that still lives. How do we find these numbers? Rather than some mystifying explanation, just watch -- you'll catch on.

$$5x - 2y = 20 \xrightarrow{\text{times } 3} 15x - 6y = 60$$
$$3x + 7y = -29 \xrightarrow{\text{times } -5} -15x - 35y = 145$$

Add the equations: $0x - 41y = 205$

The *x*'s are gone!

Divide by −41: $\dfrac{-41y}{-41} = \dfrac{205}{-41}$

$$\underline{y = -5}$$

Now that we have the value of y, we can substitute its value of -5 into either of the two original equations to find the value of x. Using the first equation:

$$\begin{aligned} & 5x - 2(\mathbf{-5}) = 20 \\ \Rightarrow\ & 5x + 10 = 20 \\ \Rightarrow\ & 5x = 10 \\ \Rightarrow\ & \underline{x = 2} \end{aligned}$$

Therefore, the final solution to the system of equations is

$$\boxed{x = 2 \ \& \ y = -5}$$

<u>EXAMPLE 2:</u> **Solve the system:** $\mathbf{12a + 7b = 9}$
$\mathbf{-18a - 5b = 3}$

<u>Solution:</u> Notice how the numbers we choose to multiply each equation by will accomplish our goal: They make the a's disappear when the equations are added.

$$\begin{aligned} 12a + 7b = 9 &\xrightarrow{\text{times } 3} 36a + 21b = 27 \\ -18a - 5b = 3 &\xrightarrow{\text{times } 2} -36a - 10b = 6 \end{aligned}$$

<u>Add</u> the equations: $0a + 11b = 33$

Divide by 11: $\underline{b = 3}$

Placing this value of b into the first equation gives us

$$12a + 7(\mathbf{3}) = 9 \Rightarrow 12a + 21 = 9 \Rightarrow 12a = -12 \Rightarrow \underline{a = -1}$$

We now have our complete solution to the system of equations:

$$\boxed{a = -1 \ \& \ b = 3}$$

Let's use this example to learn how to check our solution. The main theme is this: The values of a and b must work in <u>both</u> of the <u>original</u> equations in order to constitute a valid solution.

1st equation:

$12a + 7b = 9$

$12(\mathbf{-1}) + 7(\mathbf{3}) \stackrel{?}{=} 9$

$-12 + 21 \stackrel{?}{=} 9$

$9 = 9$ ✓

2nd equation:

$-18a - 5b = 3$

$-18(\mathbf{-1}) - 5(\mathbf{3}) \stackrel{?}{=} 3$

$18 - 15 \stackrel{?}{=} 3$

$3 = 3$ ✓

Our final conclusion is that the values $a = -1$ and $b = 3$ work perfectly. The solution can also be written as the ordered pair $(-1, 3)$.

Homework

1. Solve each system using the Elimination Method, and be sure you practice checking your solution (your pair of numbers) in <u>both</u> of the <u>original</u> equations:

a. $\begin{aligned} 2x + y &= 5 \\ -2x + 7y &= 19 \end{aligned}$

b. $\begin{aligned} 5a - 3b &= 5 \\ 10a + 4b &= -40 \end{aligned}$

c. $\begin{aligned} -2u - 3v &= -16 \\ -7u + 8v &= -56 \end{aligned}$

d. $\begin{aligned} 7x + 12y &= -24 \\ 6x - 7y &= 14 \end{aligned}$

e. $\begin{aligned} 3m - 2n &= 34 \\ -6m + n &= -62 \end{aligned}$

f. $\begin{aligned} -3s - 3t &= -24 \\ 10s + 8t &= 64 \end{aligned}$

g. $\begin{aligned} 2c - 3d &= 13 \\ 5c + 6d &= -8 \end{aligned}$

h. $\begin{aligned} -5w - 4x &= -20 \\ 20w + 3x &= 15 \end{aligned}$

i. $\begin{aligned} -5x-4n &= -8 \\ 11x+6n &= -2 \end{aligned}$

j. $\begin{aligned} 2w-4a &= 6 \\ -3w+9a &= -12 \end{aligned}$

k. $\begin{aligned} 2n-3y &= -2 \\ 8n-11y &= -2 \end{aligned}$

l. $\begin{aligned} 4c+9y &= 4 \\ -5c-11y &= 12 \end{aligned}$

m. $\begin{aligned} 5g-2h &= -6 \\ 4g+2h &= 3 \end{aligned}$

n. $\begin{aligned} -4w+3h &= -1 \\ -3w+4h &= 5 \end{aligned}$

o. $\begin{aligned} -3w+4m &= 6 \\ -3w-m &= 1 \end{aligned}$

p. $\begin{aligned} 3a+3q &= 1 \\ -5a+5q &= 6 \end{aligned}$

Solutions

1. a. $x = 1, y = 3$

Complete Check:

$2x + y = 5$	$-2x + 7y = 19$
$2(\mathbf{1}) + \mathbf{3} = 5$	$-2(\mathbf{1}) + 7(\mathbf{3}) = 19$
$2 + 3 = 5$	$-2 + 21 = 19$
$5 = 5$ ✓	$19 = 19$ ✓

b. $a = -2, b = -5$

c. $u = 8, v = 0$

d. $x = 0, y = -2$

e. $m = 10, n = -2$

f. $s = 0, t = 8$

g. $c = 2, d = -3$

h. $w = 0, x = 5$

i. $x = -4, n = 7$

j. $w = 1, a = -1$

k. $n = 8, y = 6$

l. $c = -152, y = 68$

m. $g = -\frac{1}{3}, h = \frac{13}{6}$

n. $w = \frac{19}{7}, h = \frac{23}{7}$

o. $w = -\frac{2}{3}, m = 1$

p. $a = -\frac{13}{30}, q = \frac{23}{30}$