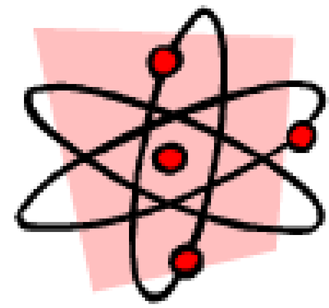
CH 3 – USING FORMULAS

❑ INTRODUCTION



In 1905, while working in a patent office in Bern, Switzerland, Albert Einstein published his statement describing the equivalence of mass and energy:

> *The energy contained in an object is equal to the mass of the object times the square of the speed of light.*

That's one way to say it. But sometimes English is not the best way to say something. In Algebra we would write this statement as

$$E = mc^2$$

[where c represents the *speed of light*, about 186,000 miles per second]

Can you see the advantage of Algebra over English?

❑ IMPORTANT TERMS

We begin with some basic terminology and notation:

Sum The *sum* of two quantities is the result of **adding** them. For example, the sum of the constants 20 and 4 is 24, and the sum of the variables x and y is written $x + y$.

Difference The *difference* of two quantities is the result of **subtracting** them. For instance, the difference of 20 and 4 is 16, and the difference of x and y is written $x - y$.

Product The *product* of two quantities is the result of **multiplying** them. We can say that 80 is the product of 20 and 4. The product of x and y can be written in any of the following ways:

$x \times y$ $\quad x \cdot y$ $\quad x(y)$ $\quad (x)y$ $\quad (x)(y)$ $\quad xy$ ⟵ the best way

In each product of x with y, both the x and the y are called **factors**, while the answer is called the **product**. And you might know that a computer would expect to see $x * y$ if you wanted the computer to find the product of x and y.

Quotient The *quotient* of two quantities is the result of **dividing** them. For example, the quotient of 20 and 4 is 5. The quotient of x and y can be written in a variety of ways:

$y\overline{)x}$	This is the classic "long division" sign from elementary school, but it will also be used in Algebra.
$x \div y$	This is the standard symbol displayed on most calculators.
x/y	This is how division is expressed in spreadsheets and computer languages.
$\frac{x}{y}$	This, of course, is the standard fraction way of showing division, and for our algebra course, it's the best way.

Square The *square* of a number is the result of multiplying that number by itself. For instance, the square of 7 is 49. The square of x is written x^2, and means $x \cdot x$, or xx. It is read "x squared," or "x to the second power," or simply "x to the second."

Parentheses When you need to make clear that a certain operation must be done before another operation, use parentheses. For example, if the sum of 7 and 8 is to be multiplied by 3, we can write

$$3(7 + 8) = 3(15) = 45$$

Homework

1. Calculate (evaluate) each expression:

a. $34 + 66$	b. $32 - 20$	c. 20×3	d. $12 \cdot 4$
e. $5(7)$	f. $(3)9$	g. $(17)(16)$	h. $25\overline{)600}$
i. $38 \div 2$	j. $3 \div 12$	k. $18/2$	l. $\frac{34}{17}$
m. $\frac{8}{10}$	n. $3(4 + 5)$	o. $6(10 - 3)$	p. 12^2

2. Computers use the asterisk, *****, to signify multiplication. Calculate 2 * 3 * 4.

3. a. What is the sum of 19 and 6?
   b. What is the sum of a and b?
   c. What is the difference of 50 and 20?
   d. What is the difference of Q and R?
   e. What is the product of 9 and 10?
   f. What is the product of u and w?
   g. What is the quotient of 55 and 11?
   h. What is the quotient of m and n?
   i. What is the square of 7?
   j. What is the square of x?
   k. What is 15 squared?
   l. What is 11 to the second power?

❑ *CONVERTING MILES AND KILOMETERS*

In almost every country of the world, large distances are measured not in miles, but in the metric system unit called the **kilometer** (accent on either the first or second syllable). One meter is a little longer than a yard, and a kilometer is 1000 meters. What we need is a pair of formulas we can use to convert between miles and kilometers. Here they are (they're actually just good approximations), where "mi" represents *miles* and "km" represents *kilometers*:

$$\text{km} = 1.6 \times \text{mi}$$

$$\text{mi} = \frac{\text{km}}{1.6}$$

Suggestion: Ask your teacher whether you need to memorize the formulas in this chapter.

EXAMPLE 1:

A. **Convert 7 miles to kilometers.**

The hardest part of this problem is determining which of the two formulas to use. Because of the way they are written, the first formula tells us how to find kilometers if we know miles, while the second formula tells us how to find miles if we know kilometers. Since the first formula starts with km, and since km is what the question is asking for, we use the first formula.

$$\text{km} = 1.6 \times \text{mi} = 1.6 \times 7 = \mathbf{11.2}$$

Therefore, 7 miles = **11.2 kilometers**

B. **Convert 13.9 kilometers to miles, and round your answer to the hundredths place.**

We're trying to find miles in this problem; since the second formula starts with mi, it's the appropriate formula to use.

$$\text{mi} = \frac{\text{km}}{1.6} = \frac{13.9}{1.6} = 8.7 \text{ (rounded off)}$$

Thus, 13.9 kilometers = **8.7 miles** (approximately)

Homework

4. Each of the following distances is given in miles. Convert to kilometers, and round to the tenths place:

a. 5 mi b. 29 mi c. 12.5 mi d. 0.23 mi e. 1.07 mi

5. Each of the following distances is given in kilometers. Convert to miles, and round to the tenths place:

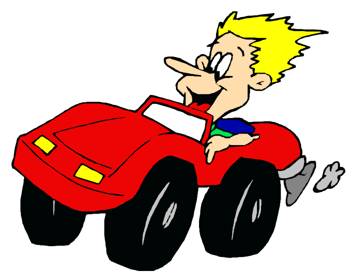
a. 12 km b. 400 km c. 17.3 km d. 0.88 km e. 0.09 km

❑ *KINETIC ENERGY*

EXAMPLE 2: **The energy an object possesses by virtue of its motion is called *kinetic energy*, and is given by the formula**

$$\boldsymbol{K = \frac{1}{2}mv^2}$$

where m is the mass and v is the velocity of the object. Find the kinetic energy of a car which has a mass of 1250 kg (kilograms) and a velocity of 30 m/s (meters per second).



Solution: The formula for kinetic energy is

$$K = \frac{1}{2}mv^2$$

Placing the given values of m and v into the formula gives:

$$K = \frac{1}{2}(1250)(30^2)$$

$$\Rightarrow \quad K = \frac{1}{2}(1250)(900)$$

If *m* is measured in kg and *v* is measured in m/s, then kinetic energy is measured in ***joules***, named after James Prescott **Joule.**

$$\Rightarrow \quad K = (625)(900) \qquad \left(\tfrac{1}{2}(1250) \text{ is } 1250 \div 2\right)$$

$$\Rightarrow \quad K = 562{,}500$$

Therefore, the kinetic energy of the car is

562,500 joules

Homework

6. Find the kinetic energy (in joules, J) given the mass (in kg) and the velocity (in m/s):

a. $m = 14;\ v = 5$

b. $m = 2;\ v = 10$

c. $m = 5;\ v = 1$

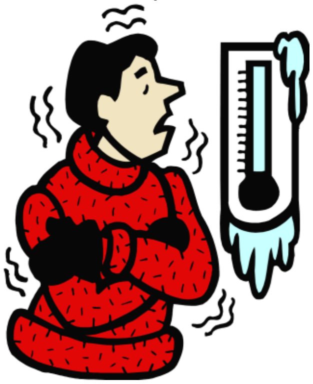
d. $m = 30;\ v = 12$

❑ *Converting Temperatures*

Consider a job announcement for prospective employees wishing to work at the new office in Victoria, British Columbia (Canada). People relocating usually want to know what kind of climate the new area has. In your research you have found the following data for average daily highs and lows for each of the four seasons in Victoria.

	Average High	Average Low
Fall	14°C	4°C
Winter	10°C	–6°C
Spring	22°C	16°C
Summer	30°C	18°C

Seems like a pretty chilly climate in Victoria -- putting data like this in an announcement isn't likely to produce many job applicants. But look closely; these temperatures are given in Celsius (in the past, called centigrade), not in Fahrenheit, the scale with which most of us are familiar. So it's clear we need to translate from Celsius to Fahrenheit before we compose the announcement.



On the ***Fahrenheit*** scale water freezes at 32°F and boils at 212°F. These numbers are a bit awkward, so the ***Celsius*** scale was created to make things easier -- on this scale, the freezing point of water is assigned the number 0°C and the boiling point is declared to be 100°C.

What we need now is a conversion formula -- that is, a formula which allows us to translate from Celsius to Fahrenheit, so that we can convert the given table to a table Americans will understand. For all we know, the climate could be even chillier than the table suggests. Here's the formula we need, and will be derived later in the course.

$$F = 1.8C + 32$$

This formula tells us that to find the Fahrenheit temperature, we need to perform two steps:

1) multiply the Celsius temperature by 1.8
2) add 32 to that result

<u>EXAMPLE 3:</u> **The average high temperature in Victoria during the Fall is 14°C. Convert this temperature to Fahrenheit.**

<u>Solution:</u> We state the relevant formula, plug in the given values, and find the value of the unknown.

$F = 1.8C + 32$	(the Celsius-to-Fahrenheit formula)
$F = 1.8(\mathbf{14}) + 32$	(plug in the 14°C that is given)
$F = 25.2 + 32$	(multiply 1.8 by 14)
$F = 57.2$	(add 25.2 and 32)

Thus, the equivalent temperature is $\boxed{57.2°\text{F}}$

So we have a formula which converts a temperature in Celsius degrees to one in Fahrenheit degrees. Now we need to <u>reverse</u> the process -- we need a formula that will convert Fahrenheit degrees into Celsius degrees. Here it is:

$$C = \frac{F - 32}{1.8}$$

This formula tells us that to find the Celsius temperature, we need to perform two steps:

1) subtract 32 from the Fahrenheit temperature

2) divide that result by 1.8

EXAMPLE 4: **Convert a room temperature of 72°F to Celsius.**

Solution: Using the Fahrenheit-to-Celsius formula, we write

$$C = \frac{F-32}{1.8} \qquad \text{(the Fahrenheit-to-Celsius formula)}$$

$$C = \frac{\mathbf{72}-32}{1.8} \qquad \text{(plug in the 72°F that is given)}$$

$$C = \frac{40}{1.8} \qquad \text{(do the top first)}$$

$$C = 22.2222\ldots \qquad \text{(and then divide 40 by 1.8)}$$

Therefore, when rounded to the nearest tenth, room temperature is about

22.2°C

Homework

7. Use the two formulas to perform the temperature conversions:

a. Convert 0°C to °F.   b. Convert 100°C to °F.

c. Convert 32°F to °C.   d. Convert 212°F to °C.

How could you have predicted the answers to these questions without using the formulas?

8. Convert the given temperature to the other scale, and round to the tenths place:

a. 100°F	b. 123°F	c. 35°F	d. 451°F
e. 14°C	f. 200°C	g. 24°C	h. 98°C
i. 50.7°F	j. 123.9°C	k. 43.5°F	l. 1000°C

Trivia Question: One of these problems is part of the title of a classic science-fiction novel. Do you know the title and author?

9. Convert body temperature, 98.6°F, to °C. Give a Celsius temperature which would represent a serious fever.

Review Problems

10. Calculate each expression:

a. $82 + 28$ b. $100 - 17$ c. $(55)(10)$ d. $\frac{88}{44}$

e. $7(7 + 3)$ f. $10(12 - 2)$ g. 9^2 h. $1^2 + 1^2$

11. a. What is the sum of 90 and 10?

b. What is the sum of x and y?

c. What is the difference of 30 and 20?

d. What is the difference of A and B?

e. What is the product of 30 and 15?

f. What is the product of g and h?

g. What is the quotient of 100 and 50?

h. What is the quotient of c and x?

i. What is the square of 20?

j. What is the square of the quantity $a + b$?

12. a. Convert 23.9 miles to kilometers.

    b. Convert 1000 kilometers to miles.

13. A student with a mass of 60 kg, who hasn't been studying his algebra, has been flung across the campus with a velocity of 7 m/s. Find the kinetic energy of this poor sap.

14. a. Convert 123.7°F to °C.

    b. Convert 0.5°C to °F.

15. Air pressure at sea level is 14.7 pounds per square inch (psi). When underwater, the water pressure increases by 14.7 psi for each additional 33 ft of depth. The formula for calculating the total pressure, P, at a depth, d, is given by

$$P = 0.445d + 14.7 \qquad [d \text{ is feet; } P \text{ is psi}]$$

    a. Confirm that at a depth of 0 ft (at the surface of the water), the pressure is 14.7 psi.

    b. Compute the pressure at a depth of 100 ft.

Solutions

1.

a.	100	b.	12	c.	60	d.	48
e.	35	f.	27	g.	272	h.	24
i.	19	j.	$\frac{1}{4}$, or 0.25	k.	9	l.	2
m.	$\frac{4}{5}$, or 0.8	n.	27	o.	42	p.	144

2. 24

3. a. $19 + 6 = 25$; the sum of 19 and 6 could be written $6 + 19$, since adding can be done in either order, but writing $19 + 6$ is consistent with the order of the numbers in the phrase "sum of 19 and 6."

b. $a + b$ (or $b + a$ if you insist)

c. $50 - 20 = 30$; turning the 50 and the 20 around will <u>not</u> produce the same answer. As we'll see later, $20 - 50 = -30$.

d. $Q - R$; just write the letters in the order in which they appear in the phrase.

e. $9 \cdot 10 = 90$; again, I have to admit that the product of 9 and 10 could also be written as $10 \cdot 9$, but don't do it!

f. The product of u and w, in that order, is uw, which can also be written $u(w)$ or $u \cdot w$

g. $\frac{55}{11}$, or $55 \div 11$, both of which = 5

h. The quotient of m and n can be written in four different ways, but the best way is $\frac{m}{n}$.

i. $7^2 = 7 \times 7 = 49$

j. x^2

k. 225

l. 121

4. a. 8 km b. 46.4 km c. 20 km
d. 0.4 km e. 1.7 km

5. a. 7.5 mi b. 250 mi c. 10.8 mi
d. 0.6 mi e. 0.1 mi

6. a. 175 J b. 100 J c. 2.5 J d. 2160 J

7. a. 32°F b. 212°F c. 0°C d. 100°C

8. a. 37.8°C b. 50.6°C c. 1.7°C d. 232.8°C
e. 57.2°F f. 392°F g. 75.2°F h. 208.4°F
i. 10.4°C j. 255°F k. 6.4°C l. 1832°F

9. 37°C; a serious fever might be 40°C.

10. a. 110 b. 83 c. 550 d. 2
e. 70 f. 100 g. 81 h. 2

11. a. 100 b. $x + y$ c. 10 d. $A - B$ e. 450
f. gh g. 2 h. $\frac{c}{x}$ i. 400 j. $(a + b)^2$

12. a. 38.24 km b. 625 mi

13. 1470 J

14. a. 50.9°C b. 32.9°F

15. a. $P = 0.445d + 14.7$
$P = 0.445(\mathbf{0}) + 14.7$
$P = 0 + 14.7$
$P = 14.7$ ✓ It's true; at a depth of 0, the pressure is 14.7 psi.

b. $P = 59.2$ psi

❑ *TO ∞ AND BEYOND*

A. What happens to the *kinetic energy* of an object if the mass is doubled?

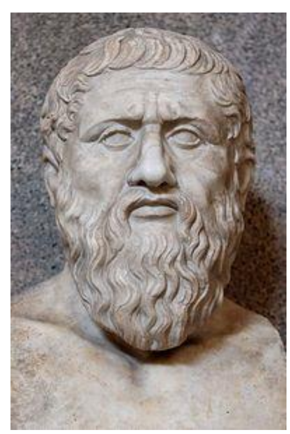
B. What happens to the kinetic energy of an object if the velocity is tripled?

C. What happens to the kinetic energy of an object if the mass is tripled and the velocity is quadrupled?

D. Even though we haven't explored negative numbers yet in this class, our formulas could be used to convert 300° below zero on the Celsius scale to Fahrenheit. Do some research to determine why, in actuality, this problem makes no sense.

E. Use the formula for water pressure and depth to calculate the depth of a diver if the pressure is 459.7 psi.

"The educated differ from the uneducated as much as the living from the dead."



Αριστοτλε (Aristotle)