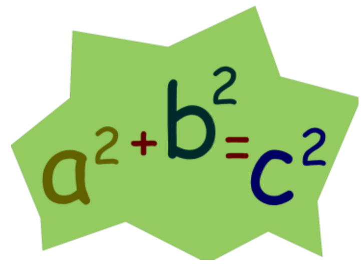
CH 19 – SOLVING FORMULAS

❑ INTRODUCTION



Solving equations such as $2x + 7 = 20$ is obviously the cornerstone of algebra. But in science, business, and computers it is also necessary to solve equations that might have a variety of letters in them. For example, if you wanted to find the temperature in the gas law $PV = nRT$, you might have to solve this equation for T.

A ***formula*** (called a *literal equation* by many) is just an equation with two or more variables in it, and is designed to accomplish something specific. The equation for the perimeter of a rectangle, $P = 2l + 2w$, is an example of a formula. If we needed to solve for the width in this rectangle formula, we would have to isolate the variable w using the same equation-solving techniques we've just studied.

❑ REVIEW OF SOLVING EQUATIONS

Some students find the solving of formulas to be very abstract, while others consider it much easier than solving regular equations. Let's begin this chapter with a quick review of solving one-step equations.

To solve the equation $n + 3 = 10$, we recognize that 3 has been added to the unknown n. To remove the 3 -- that is, to undo the addition -- we subtract 3 from each side of the equation. This isolates, or solves for, the n, and we get $n = 7$.

Similarly, to solve the equation $y - 10 = 27$, we add 10 to each side of the equation, resulting in a solution of $y = 37$.

To isolate the variable in the equation $9w = 45$, we notice that multiplication binds the 9 with the w. To remove the 9, therefore, we divide each side of the equation by 9, yielding a value of 5 for w.

For our last review example in this section, to solve the equation $\frac{x}{12} = 24$ we are obliged to multiply each side of the equation by 12, since multiplying undoes dividing. And so $x = 288$.

In the following sections we also "isolate" variables, but now we have to remove other variables (not just numbers) in order to isolate the one we want. Since there's no arithmetic involved -- just algebra concepts -- you can see why some students think this is not so bad, after all. Perhaps with a little practice you'll feel the same way.

❑ *ONE-STEP EXAMPLES*

EXAMPLE 1: **Solve for x: $x + a = b$**

Solution: To isolate the x we need to remove the a. Since the a is being added to the x, we remove it by subtracting it from each side of the equation, thus isolating the x:

$$x + a = b \quad \text{(the original formula)}$$

$$\Rightarrow \quad x + a - \mathbf{a} = b - \mathbf{a} \quad \text{(subtract } a \text{ from each side)}$$

$$\Rightarrow \quad \boxed{x = b - a} \quad \text{(simplify)}$$

EXAMPLE 2: **Solve for u: $u - w = A$**

Solution: To isolate the u, we need to remove the w. Since the w is being subtracted from the u, we remove it by adding w to each side of the equation:

$$u - w = A \quad \text{(the original formula)}$$

$$\Rightarrow \quad u - w + \mathbf{w} = A + \mathbf{w} \quad \text{(add } w \text{ to each side)}$$

$$\Rightarrow \quad \boxed{u = A + w} \quad \text{(simplify)}$$

EXAMPLE 3: **Solve for n: $cn = R$**

Solution: To isolate the n we need to remove the c. Since the operation between the c and the n is multiplication, we remove the c by dividing each side of the equation by c:

$cn = R$ (the original formula)

$\Rightarrow \frac{cn}{c} = \frac{R}{c}$ (divide each side by c)

$\Rightarrow \boxed{n = \frac{R}{c}}$ (simplify)

EXAMPLE 4: **Solve for W: $\frac{W}{a} = -x$**

Solution: To isolate the W we need to remove the a. Since W is being divided by a, we can remove the a by multiplying each side of the equation by a:

$\frac{W}{a} = -x$ (the original formula)

$\Rightarrow \frac{W}{a}[a] = -x[a]$ (multiply each side by a)

$\Rightarrow \frac{W}{\cancel{a}}[\cancel{a}] = -ax$ (cross-cancel the a's)

$\Rightarrow \boxed{W = -ax}$ (simplify)

EXAMPLE 5: **Solve for x: $x(y - z) = Q$**

Solution: To isolate the x we need to remove the quantity $y - z$. We ask, What is the operation connecting the x with the $y - z$? It's multiplication, so we use division to reverse the operation and thus isolate the x:

$$x(y - z) = Q \quad \text{(the original formula)}$$

$$\Rightarrow \frac{x(y-z)}{\boldsymbol{y-z}} = \frac{Q}{\boldsymbol{y-z}} \quad \text{(divide each side by } y - z\text{)}$$

$$\Rightarrow \boxed{x = \frac{Q}{y-z}} \quad \text{(simplify)}$$

EXAMPLE 6: **Solve for the temperature T in the gas law $PV = nRT$ mentioned in the Introduction.**

Solution:

$$PV = nRT \quad \text{(the original gas law)}$$

$$\Rightarrow \frac{PV}{\boldsymbol{nR}} = \frac{nRT}{\boldsymbol{nR}} \quad \text{(divide each side by } nR\text{)}$$

$$\Rightarrow \frac{PV}{nR} = T \quad \text{(simplify the right side)}$$

$$\Rightarrow \boxed{T = \frac{PV}{nR}} \quad \text{(reverse the equation)}$$

EXAMPLE 7: **Solve each formula for x:**

A. $\dfrac{x}{a} = b + c$

Now comes a new issue -- we know that we need to multiply each side of the equation by a (in order to isolate the x). Multiplying the left side by a is easy; it cross-cancels with the a, leaving just x, the unknown. But how do we indicate that the right-hand side of the equation, the quantity $b + c$, all of it, must be multiplied by a? We put parentheses around the quantity $b + c$, that's how:

note the parentheses

$$\frac{x}{a}\left[\boldsymbol{a}\right] = (b+c)\left[\boldsymbol{a}\right]$$

Simplifying the left side of the equation gives x. Simplifying the right side gives simply $(b + c)a$. Thus,

$$x = (b + c)a,$$

or, by the commutative property for multiplication,

$\boldsymbol{x = a(b + c)}$, and we're done.

Note: Another way to write the final solution is to distribute and get $x = ab + ac$. But this is not necessary, since the goal of this chapter is to learn how to isolate variables, not simplify answers.

B. $\dfrac{x}{y-z} = w$

Again, we are trying to isolate the x, and again we will accomplish this by multiplying each side of the equation by the denominator, in this case $y - z$. Remembering the use of parentheses (or brackets), we get

$\dfrac{x}{y-z}\left[\boldsymbol{y-z}\right] = w\left[\boldsymbol{y-z}\right]$, and our final answer is

$\boldsymbol{x = w[y - z]}$, or $\boldsymbol{x = w(y - z)}$

C. $\dfrac{x}{c-e} = a+u$

Multiplying each side of the equation by $c - e$ produces

$$\frac{x}{c-e}\left[\boldsymbol{c-e}\right] = (a+u)\left[\boldsymbol{c-e}\right]$$

$$\Rightarrow \quad \boldsymbol{x = (a + u)(c - e)}$$

Homework

1. Solve each formula for *x*:

a. $x + b = a$ b. $cx = d$ c. $x - y = z$

d. $\dfrac{x}{L} = T$ e. $c + x = m$ f. $xy = z$

g. $-R + x = w$ h. $\dfrac{x}{-a} = -b$ i. $x - m = n$

j. $x + a = b$ k. $-b + x = W$ l. $\dfrac{x}{y} = -z$

m. $a + x = w$ n. $x - ab = c$ o. $x + bc = d$

2. Solve each formula for *a*:

a. $a(b - c) = d$ b. $a(Q + R) = S$ c. $(b_1 + b_2)a = A$

d. $abc = d$ e. $mag = P$ f. $a(r_1 + r_2) = T$

g. $bca = M$ h. $a(x - y) = z$ i. $a(w - u) = n$

j. $axy = z$ k. $W = abc$ l. $Q = a(g - h)$

3. Solve each formula for *y*:

a. $\dfrac{y}{a} = b + c$ b. $\dfrac{y}{c} = d - e$ c. $\dfrac{y}{x} = x + a$

d. $\dfrac{y}{t} = b + c - d$ e. $\dfrac{y}{c} = x - w + z$ f. $\dfrac{y}{x - z} = A$

g. $\dfrac{y}{w + 7} = c$ h. $\dfrac{y}{a - b} = Q$ i. $\dfrac{y}{a - b + c} = w$

j. $\dfrac{y}{x + g - h} = k$ k. $\dfrac{y}{L + M - N} = P$ l. $\dfrac{y}{u + w} = x + z$

m. $\dfrac{y}{a - R} = Q + T$ n. $\dfrac{y}{m - n} = g - h$

o. $\dfrac{y}{a + b - c} = w + u$ p. $\dfrac{y}{x + w} = a + b + c$

q. $\dfrac{y}{u + w - x} = d - e + h$

❑ TWO-STEP EXAMPLES

Let's review a standard two-step equation. If we solve the equation $3x - 17 = 82$ for x, we remember the dilemma of deciding which to rid ourselves of first, the 3 or the 17. We agreed that reversing the Order of Operations was the secret. So, from the x's point of view, it was multiplied by 3 first, and then 17 was subtracted. Reversing this sequence means we first add 17 to each side, giving $3x = 99$; then we divide each side by 3 to get the value $x = 33$. Well, it's even easier with letters [I hope!].

EXAMPLE 8: **Solve for y: $ay - b = c$**

Solution: The Order of Operations tells us that first the a and the y were multiplied, and then b was subtracted. To isolate the y we need to reverse the Order of Operations: Add b to each side, and then divide each side by a:

$ay - b = c$ (the original formula)

$\Rightarrow \quad ay - b + \mathbf{b} = c + \mathbf{b}$ (add b to each side)

$\Rightarrow \quad ay = c + b$ (simplify)

$\Rightarrow \quad \dfrac{ay}{\mathbf{a}} = \dfrac{c+b}{\mathbf{a}}$ (divide each side by a)

$\Rightarrow \quad \boxed{y = \dfrac{c+b}{a}}$ (simplify)

Note: It's common to use alphabetical order when writing variables, so the answer can also be written $y = \dfrac{b+c}{a}$.

EXAMPLE 9: **Solve for x:** $\boldsymbol{\frac{x}{a} + e = d}$

Solution: The x has been divided by a, and then e has been added. Reversing these operations produces the following steps:

$$\frac{x}{a} + e = d \qquad \text{(the original formula)}$$

$$\Rightarrow \quad \frac{x}{a} + e - \boldsymbol{e} = d - \boldsymbol{e} \qquad \text{(subtract } e \text{ from each side)}$$

$$\Rightarrow \quad \frac{x}{a} = d - e \qquad \text{(simplify)}$$

Now recall from the previous section how we isolate the x. We need to multiply each side of the equation by a, and we need to put parentheses around the quantity $d - e$ when we do.

$$\Rightarrow \quad \frac{x}{a}\left[\boldsymbol{a}\right] = (d-e)\left[\boldsymbol{a}\right] \qquad \text{(multiply each side by } a \text{ -- notice the parentheses)}$$

$$\Rightarrow \quad \boxed{x = a(d-e)} \qquad \text{(simplify)}$$

Homework

4. Solve each formula for x:

a. $ax + b = c$ b. $cx - d = R$ c. $a + bx = n$

d. $m - nx = Q$ e. $\frac{x}{b} - e = h$ f. $\frac{x-b}{c} = a$

g. $ab + xy = cd$ h. $\frac{x+b}{d} = p$ i. $\frac{x}{a} + k = L$

j. $ax - d = c$ k. $cx + d = e$ l. $a - bx = n$

m. $t + nx = R$ n. $\frac{x}{b} + h = h$ o. $\frac{x+c}{c} = a$

p. $2a + xy = cd$ q. $\frac{x+7}{e} = \pi$ r. $\frac{x}{b} - j = N$

5. Solve each formula for y:

Example: Solve for y: $3x + 5y + 1 = 0$

Solution:

$5y + 1 = -3x$ (subtract $3x$ from each side)

$5y = -3x - 1$ (subtract 1 from each side)

$y = \frac{-3x-1}{5}$ (divide each side by 5)

a. $y + 2x = 10$

b. $y - 3x = -13$

c. $y + 4x - 12 = 0$

d. $3x + y = -7$

e. $-4x + y - 1 = 0$

f. $-3x + y = 7$

g. $-2x + y + 1 = 0$

h. $3x + 2y = 10$

i. $2x - 7y = 14$

j. $-8x - 2y + 7 = 0$

k. $-3x + 8y = 1$

l. $9x - 5y + 2 = 0$

❑ *MULTI-STEP EXAMPLES*

EXAMPLE 10: **Solve for n:** $\frac{Tn+a}{b} = c$

Solution: To solve this formula for n, we must remove the T, the a, and the b. At each step determine the final operation, and perform the reverse operation.

Start with the original problem: $\frac{Tn+a}{b} = c$

The final operation is division by b. Remove the b by multiplying each side of the equation by b: $b\left(\frac{Tn+a}{b}\right) = b(c)$

Simplify each side: $Tn + a = bc$

Now the final operation is the addition of the a. Remove the a by subtracting it from both sides: $Tn = bc - a$

Now divide both sides by T: $$\frac{Tn}{T} = \frac{bc-a}{T}$$

Simplify each side, and we're done: $$\boxed{n = \frac{bc-a}{T}}$$

EXAMPLE 11: **Solve for x:** $\boldsymbol{\frac{nx-w}{y+z} = e-f}$

Solution: This formula has the same structure as the one in the previous example, except that the denominator and the quantity on the right consist of two terms instead of one. The only thing to remember is to use parentheses wherever appropriate.

$$\frac{nx-w}{y+z} = e-f \qquad \text{(original formula)}$$

$$\Rightarrow \quad \frac{nx-w}{y+z}\left[y+z\right] = \left(e-f\right)\left[y+z\right] \qquad \text{(multiply each side by } y+z\text{)}$$

$$\Rightarrow \quad nx-w = (e-f)(y+z) \qquad \text{(simplify)}$$

$$\Rightarrow \quad nx = (e-f)(y+z)+w \qquad \text{(add } w \text{ to each side)}$$

$$\Rightarrow \quad \boxed{x = \frac{(e-f)(y+z)+w}{n}} \qquad \text{(divide each side by } n\text{)}$$

EXAMPLE 12: **Solve for R:** $\boldsymbol{\frac{aR-b}{n}+d = Q}$

Solution: This might be the appropriate place to show you another approach to solving complicated formulas. The eventual steps we will take will be exactly as we've done in the previous examples. But some students like to see the Order of Operations explicitly numbered; this helps them see what operation to undo at each stage of the problem.

Pretend you were R, the unknown. Let's list exactly what's been done to you, using the Order of Operations:

1. you were multiplied by a
2. b was then subtracted
3. the whole thing was then divided by n
4. last, d was added on

To untangle this mess, we need to reverse the Order of Operations, and reverse the operation at each stage of the Order of Operations:

1. subtract d
2. multiply by n
3. add b
4. divide by a

The original formula: $\dfrac{aR-b}{n}+d = Q$

1. Subtract d from each side: $\dfrac{aR-b}{n}+d-d = Q-d$

   Simplify: $\dfrac{aR-b}{n} = Q-d$

2. Multiply each side by n: $\dfrac{aR-b}{n}[n] = (Q-d)[n]$

   Simplify: $aR-b = n(Q-d)$

3. Add b to each side: $aR-b+b = n(Q-d)+b$

   Simplify: $aR = n(Q-d)+b$

4. Divide each side by a: $\dfrac{aR}{a} = \dfrac{n(Q-d)+b}{a}$

   We made it! $\boxed{R = \dfrac{n(Q-d)+b}{a}}$

Homework

6. Solve each formula for x:

a. $\frac{x+a}{b} = c$

b. $\frac{3x-a}{d} = w$

c. $\frac{Tx+\pi}{\pi} = A$

d. $\frac{x+a}{c+d} = R$

e. $\frac{bx-c}{y} + L = M$

f. $\frac{nx+m}{a-b} = y-w$

g. $\frac{ax+u-w}{b} = c$

h. $\frac{xy-a+b}{T} = Z$

i. $\frac{ax-c+d}{L} = T$

j. $\frac{tx+a}{c} = b$

k. $\frac{ax+b}{y+z} = g-h$

l. $\frac{ax-b}{Q} + c = d$

m. $\frac{7x+u-w}{z} = a$

n. $\frac{ax}{b-c} = d$

o. $\frac{hx+p}{p-q} = a+b$

p. $\frac{x-a}{b} = d$

q. $\frac{\pi x+a}{d} = w$

r. $\frac{Tx-e}{e} = b$

s. $\frac{x-a}{c+d} = R$

t. $\frac{bx+c}{c} - L = M$

u. $\frac{nx-n}{a+b} = y+w$

v. $\frac{ax-u+w}{c} = d$

w. $\frac{xy+y+z}{S} = A$

x. $\frac{ax+b-L}{Q} = T$

y. $\frac{wx-a}{c} = b$

z. $\frac{ax-b}{y-z} = g+h$

❑ *SOLVING REAL FORMULAS*

Homework

7. Solve for m in the kilometers/miles formula $k = 1.61m$.

8. Solve for k in the kilometers/miles formula $m = \frac{k}{1.61}$.

9. Solve for C in the temperature formula $F = 1.8C + 32$.

10. Solve for F in the temperature formula $C = \frac{F-32}{1.8}$.

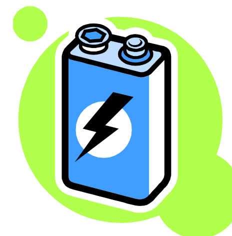
11. Solve for L in the assets/liabilities/capital formula $A = L + C$.

12. Solve for R in the profit/revenue/expense formula $P = R - E$.

13. Solve for t in the distance/rate/time formula $d = rt$.

14. Solve for m in the density/mass/volume formula $d = \frac{m}{V}$.

15. Solve for S in the average formula $A = \frac{S}{n}$.



16. Solve for i in Ohm's Law $V = iR$.

17. Solve for R in the gas law $PV = nRT$.

18. Solve for m in the potential energy formula $P = mgh$.

19. Solve for m in Einstein's equation $E = mc^2$.

20. Solve for l in the perimeter formula $P = 2l + 2w$.

21. Solve for π in the circle area formula $A = \pi r^2$.

❑ *FINAL NOTE*

Here's a parting comment regarding the solving of any formula:

The final answer can <u>never</u> contain
the variable you're solving for.

For example, a final solution such as

$$x = ab + cx$$

is outright insanity. Here's why: This solution says that to <u>determine</u> the value of <u>x</u>, we need to <u>know</u> the values of a, b, c, <u>and x</u>! A fatal case of circular reasoning, indeed.

And now for something ironic: In computer languages the statement $x = x + 3$ is perfectly valid and tremendously useful (it merely increases the value of x by 3.) But it is <u>not</u> an equation, and we're not trying to solve anything, so it doesn't violate the rule above.

Review Problems

22. Solve for w in the perimeter formula $P = 2l + 2w$.

23. Solve for w in the area formula $A = lw$.

24. Solve for r in the circumference formula $C = 2\pi r$.

25. Solve for d in the radius formula $r = \frac{d}{2}$.

26. Solve for A in the formula $\frac{A}{r^2} = \pi$.

27. Solve each formula for x:

a. $x - c = d$ b. $2x + b = R$ c. $abx = c$

d. $\frac{x}{u} = N$ e. $x(y + z) = a$ f. $\frac{x}{n} = c - d$

g. $\frac{x}{a+b} = m - n$ h. $\frac{x}{c-Q} = c + Q$ i. $\frac{x}{R} = a - b + c$

j. $x(b_1 + b_2) = A$ k. $\frac{x}{a} - e = m$ l. $\frac{x+a}{b} = y$

m. $9x - 7y + 13 = 0$

28. Your neighbor solves the formula $ax + bx = c$ for x, and comes up with a solution of $x = \frac{c - bx}{a}$. Your comments, please.

Solutions

1. a. $x = a - b$ b. $x = \frac{d}{c}$ c. $x = z + y$, or $x = y + z$

d. $x = TL$, or $x = LT$ e. $x = m - c$ f. $x = \frac{z}{y}$

g. $x = W + R$ h. $x = ab$ i. $x = n + m$

j. $x = b - a$ k. $x = W + b$ l. $x = -zy$, or $x = -yz$

m. $x = w - a$ n. $x = c + ab$ o. $x = d - bc$

2. a. $a = \frac{d}{b-c}$ b. $a = \frac{S}{Q+R}$ c. $a = \frac{A}{b_1+b_2}$

d. $a = \frac{d}{bc}$ e. $a = \frac{P}{mg}$ f. $a = \frac{T}{r_1+r_2}$

g. $a = \frac{M}{bc}$ h. $a = \frac{z}{x-y}$ i. $a = \frac{n}{w-u}$

j. $a = \frac{z}{xy}$ k. $a = \frac{W}{bc}$ l. $a = \frac{Q}{g-h}$

3. a. $y = a(b+c)$ b. $y = c(d-e)$ c. $y = x(x+a)$

d. $y = t(b+c-d)$ e. $y = c(x-w+z)$ f. $y = A(x-z)$

g. $y = c(w+7)$ h. $y = Q(a-b)$ i. $y = w(a-b+c)$

j. $y = k(x+g-h)$ k. $y = P(L+M-N)$ l. $y = (x+z)(u+w)$

m. $y = (Q+T)(a-R)$ n. $y = (g-h)(m-n)$

o. $y = (w+u)(a+b-c)$ p. $y = (x+w)(a+b+c)$

q. $y = (d-e+h)(u+w-x)$

4. a. $x = \frac{c-b}{a}$ b. $x = \frac{R+d}{c}$ c. $x = \frac{n-a}{b}$

d. $x = \frac{Q-m}{-n}$ e. $x = (h+e)b$, or $x = b(h+e)$

f. $x = ac+b$ g. $x = \frac{cd-ab}{y}$ h. $x = pd-b$

i. $x = (L-k)a$, or $x = a(L-k)$ j. $x = \frac{c+d}{a}$

k. $x = \frac{e-d}{c}$ l. $x = \frac{n-a}{-b}$ m. $x = \frac{R-t}{n}$

n. $x = 0$ o. $x = ac-c$ p. $x = \frac{cd-2a}{y}$

q. $x = \pi e - 7$ r. $x = b(N+j)$, or $x = (N+j)b$

5. a. $y = 10-2x$ b. $y = -13+3x$ c. $y = 12-4x$

d. $y = -7 - 3x$ e. $y = 1 + 4x$ f. $y = 7 + 3x$

g. $y = 2x - 1$

h. $3x + 2y = 10 \Rightarrow 2y = 10 - 3x \Rightarrow y = \frac{10 - 3x}{2}$

i. $y = \frac{14 - 2x}{-7}$ j. $y = \frac{8x - 7}{-2}$ k. $y = \frac{1 + 3x}{8}$

l. $y = \frac{-2 - 9x}{-5}$

6. a. $x = bc - a$ b. $x = \frac{dw + a}{3}$ c. $x = \frac{A\pi - \pi}{T}$

d. $x = R(c + d) - a$ e. $x = \frac{y(M - L) + c}{b}$

f. $x = \frac{(y - w)(a - b) - m}{n}$ g. $x = \frac{bc + w - u}{a}$ h. $x = \frac{ZT - b + a}{y}$

i. $x = \frac{LT - d + c}{a}$ j. $x = \frac{bc - a}{t}$

k. $x = \frac{(g - h)(y + z) - b}{a}$ l. $x = \frac{(d - c)Q + b}{a}$ or $x = \frac{Q(d - c) + b}{a}$

m. $x = \frac{az + w - u}{7}$ n. $x = \frac{d(b - c)}{a}$

o. $x = \frac{(a + b)(p - q) - p}{h}$ p. $x = db + a$ q. $x = \frac{wd - a}{\pi}$

r. $x = \frac{be + e}{T}$ s. $x = R(c + d) + a$

t. $x = \frac{(M + L)c - c}{b}$ u. $x = \frac{(y + w)(a + b) + n}{n}$

v. $x = \frac{cd - w + u}{a}$ w. $x = \frac{AS - z - y}{y}$

x. $x = \frac{TQ + L - b}{a}$ y. $x = \frac{bc + a}{w}$

z. $x = \frac{(g + h)(y - z) + b}{a}$

7. $m = \frac{k}{1.61}$ **8**. $k = 1.61m$ **9**. $C = \frac{F - 32}{1.8}$

10. $F = 1.8C + 32$ **11**. $L = A - C$ **12**. $R = P + E$

13. $t = \frac{d}{r}$ **14**. $m = dV$ **15**. $S = An$

16. $i = \frac{V}{R}$

17. $R = \frac{PV}{nT}$

18. $m = \frac{P}{gh}$

19. $m = \frac{E}{c^2}$

20. $l = \frac{P-2w}{2}$

21. $\pi = \frac{A}{r^2}$

22. $w = \frac{P-2l}{2}$

23. $w = \frac{A}{l}$

24. $r = \frac{C}{2\pi}$

25. $d = 2r$

26. $A = \pi r^2$

27. a. $x = d + c$

b. $x = \frac{R-b}{2}$

c. $x = \frac{c}{ab}$

d. $x = Nu$

e. $x = \frac{a}{y+z}$

f. $x = n(c - d)$

g. $x = (m - n)(a + b)$

h. $x = (c + Q)(c - Q)$

i. $x = R(a - b + c)$

j. $x = \frac{A}{b_1 + b_2}$

k. $x = a(m + e)$

l. $x = by - a$

m. $x = \frac{7y-13}{9}$

28. When solving a formula for *x*, the only letter that can't possibly be in the final answer is *x* itself. For one thing, solving a formula for *x* means to isolate it. How isolated is it when there's an *x* on each side of the equals sign?

Second, the bogus solution is circular reasoning. According to your neighbor's solution, to find the value of *x* we would need the values of *a*, *b*, and *c*, and *x*! That is, to find the value of *x*, we would need the value of *x*. That's crazy.

❑ **To ∞ and Beyond**

Solve for x: $$\frac{\dfrac{\dfrac{ax+b}{c}-d+e}{f-g}+gh-k}{w+z}+y = n$$

“That is what learning is.
You suddenly understand something
you’ve understood all your life,
but in a new way.”

– Doris Lessing