CH 25 – COINS

❑ Introduction

Milo has 7 dimes. This means he has \$0.70 worth of dimes. We'll call the **7** the *number* of coins that Milo has, we'll call **\$0.10** (or 10¢) the *unit value*, and we'll call the **\$0.70** (or 70¢) the *total value* of Milo's coins.

If Sierra has 5 quarters, then the *number* of coins is 5, the *unit value* is \$0.25 (or 25¢), and the *total value* is \$1.25 (or 125¢).

In both examples, how was the total value calculated? By multiplying the number of coins by their unit value. For example, using Sierra's quarters:

5 quarters × \$0.25 per quarter = \$1.25

In general,

Number of Coins × Unit Value = Total Value

❑ Coin Problems

EXAMPLE 1: **Brennan has 3 times as many nickels as dimes. (That is, the number of nickels is 3 times the number of dimes.) If the total value of the coins is \$2.75, how many of each coin does Brennan have?**

Solution: The easiest way to solve these kinds of problems (so that we can avoid decimals) is to think of all money amounts in

pennies (cents). For example, instead of saying that a quarter has a value of \$0.25, we'll say that its value is 25¢. And the total value of \$2.75 in this problem will be represented by the number 275¢. So, here we go.

First we notice that we are asked for two different things: the number of nickels and the number of dimes. That's two variables we'll have to create -- let's declare n to represent the number of nickels, and let d equal the number of dimes. As you'll see, this means that we'll need two equations in order to find the values of the two variables.

Let's set up a chart to solve this problem. The central idea of the chart is that, for any given kind of coin, if we multiply the number of coins by its unit value, we get the total value of that coin. Study this chart carefully.

	Number of Coins × Unit Value = Total Value		
nickels	n	5	$5n$
dimes	d	10	$10d$
mixture	n/a	n/a	275

Since it's given that the number of nickels is 3 times the number of dimes, we get our first equation:

$$n = 3d$$

Also, the total value of the nickels ($5n$) plus the total value of the dimes ($10d$) should equal the total value of the mixture; that is,

$$5n + 10d = 275$$

Sure enough, we have two equations with two variables, just as we had predicted. And now we solve the set of two equations by rewriting the second equation, but using the fact that $n = 3d$:

$$5(\mathbf{3}\boldsymbol{d}) + 10d = 275 \qquad \text{(substitute } 3d \text{ for } n\text{)}$$

$$\Rightarrow \quad 15d + 10d = 275$$

$\Rightarrow \quad 25d = 275$

$\Rightarrow \quad d = 11$, so Brennan has 11 dimes

Since $d = 11$, it's pretty easy to calculate n:

$n = 3d = 3(\mathbf{11}) = 33$, so Brennan also has 33 nickels.

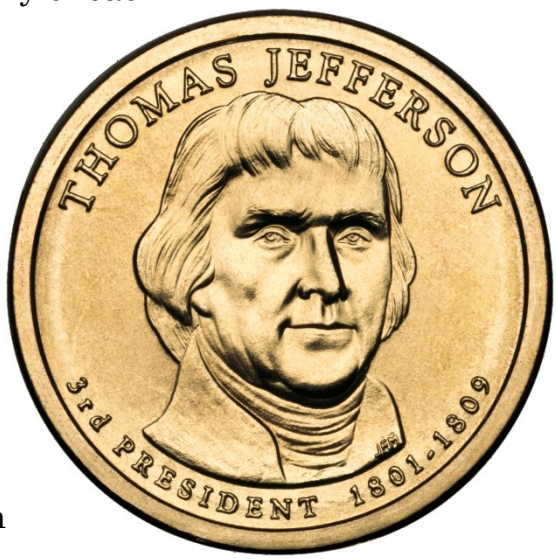
In summary, Brennan has

11 dimes and 33 nickels

Homework

1. Lisa has 7 times as many dimes as nickels. If the total value of the coins is \$3.75, how many of each coin does Lisa have?

2. Orion has 5 times as many quarters as nickels. If the total value of the coins is \$3.90, how many of each coin does Orion have?



3. Lauren has 4 times as many pennies as dimes. If the total value of the coins is \$0.98, how many of each coin does Lauren have?

4. Cal has 10 times as many nickels as quarters. If the total value of the coins is \$6.75, how many of each coin does Cal have?

EXAMPLE 2: **Alana has 3 more quarters than pennies. If the total value of the coins is $5.17, how many of each coin does Alana have?**

Solution: Letting q represent the number of quarters and p the number of pennies, here's the chart:

	Number of Coins × Unit Value = Total Value		
quarters	q	25	$25q$
pennies	p	1	$1p$
mixture	n/a	n/a	517

The first sentence of the problem, the relationship between the number of quarters and the number of pennies, can also be expressed this way: *The number of quarters is 3 more than the number of pennies.* This leads to the equation:

$$q = p + 3$$

For our second equation, we use the same logic as Example 1. The value of the quarters ($25q$) plus the value of the pennies ($1p$) should equal the total value of the coin mixture; that is,

$$25q + 1p = 517$$

Since $q = p + 3$ from the first equation, we now change the q in the second equation to $p + 3$:

$$25(\boldsymbol{p + 3}) + 1p = 517 \qquad \text{(since } q = p + 3\text{)}$$

$$\Rightarrow \quad 25p + 75 + p = 517 \qquad \text{(distribute)}$$

$$\Rightarrow \quad 26p + 75 = 517 \qquad \text{(combine like terms)}$$

$$\Rightarrow \quad 26p = 442 \qquad \text{(subtract 75 from each side)}$$

$$\Rightarrow \quad p = 17 \qquad \text{(divide each side by 26)}$$

So Alana has 17 pennies.

Thus, the number of quarters is

$$q = p + 3 = \mathbf{17} + 3 = 20$$

To conclude, Alana has

20 quarters and 17 pennies

Homework

5. Alana has 5 more quarters than nickels. If the total value of the coins is \$4.25, how many of each coin does Alana have?

6. Ben has 7 more dimes than pennies. If the total value of the coins is \$1.47, how many of each coin does Ben have?

7. Andrew has 2 more nickels than quarters. If the total value of the coins is \$5.20, how many of each coin does Andrew have?

8. Justus has 10 more pennies than dimes. If the total value of the coins is \$3.73, how many of each coin does Justus have?

EXAMPLE 3: **Maria has a total of 14 coins, some quarters and some dimes. If the total value of the coins is \$2.75, how many of each coin does Maria have?**

Solution: Since Maria has some unknown number of quarters and dimes, let's let q stand for the number of quarters and let d be the number of dimes. The following chart is very similar to the previous ones:

	Number of Coins ×	Unit Value =	Total Value
quarters	q	25	$25q$
dimes	d	10	$10d$
mixture	14	n/a	275

We're ready for our first equation:

$q + d = 14$ (14 coins altogether)

What else do we know? As before, the total value of the quarters plus the total value of the dimes must equal the total value of the coin mixture, which is 275¢:

$25q + 10d = 275$

Let's summarize what we have, and what we're looking for. We're now trying to solve two equations with two variables:

$q + d = 14$
$25q + 10d = 275$ (we want to find both q and d)

How about we solve for q in the first equation, $q + d = 14$. By subtracting d from each side of the equation, our system of equations becomes

$q = 14 - d$
$25q + 10d = 275$

Now change the q in the second equation to $14 - d$ from the first equation:

$25(14 - d) + 10d = 275$ (since $q = 14 - d$)

What has this maneuver accomplished? We now have an equation with just <u>one</u> variable in it -- this we can solve:

$\Rightarrow \quad 350 - 25d + 10d = 275$ (distribute)

$\Rightarrow \quad 350 - 15d = 275$ (combine like terms)

$\Rightarrow \quad -15d = -75$ (subtract 350 from each side)

$\Rightarrow \quad d = 5$ (divide each side by –15)

This means that Maria has 5 dimes, and since $q = 14 - d$, we calculate that the number of quarters is $14 - 5 = 9$. In summary, Maria has

9 quarters and 5 dimes

Homework

9. The total number of coins is 20, some nickels and some dimes. If the total value of the coins is $1.40, how many of each coin are there?

10. The total number of coins is 29, some quarters and some pennies. If the total value of the coins is $3.89, how many of each coin are there?

11. The total number of coins is 40, some nickels and some quarters. If the total value of the coins is $5.00, how many of each coin are there?

12. The total number of coins is 87, some dimes and some pennies. If the total value of the coins is $6.09, how many of each coin are there?

Solutions

1. 5 nickels and 35 dimes

2. 15 quarters and 3 nickels

3. 28 pennies and 7 dimes

4. 90 nickels and 9 quarters

5. 15 quarters and 10 nickels

6. 14 dimes and 7 pennies

7. 19 nickels and 17 quarters

8. 43 pennies and 33 dimes

9. 12 nickels and 8 dimes

10. 15 quarters and 14 pennies

11. 25 nickels and 15 quarters

12. 58 dimes and 29 pennies

"Any fool can know. The point is to understand."

– *Albert Einstein*