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# CH 33 – TRIANGLES AND ANGLES

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## □ Introduction

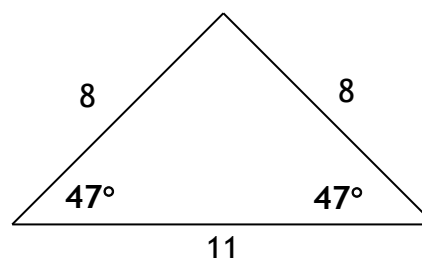
An isosceles triangle has at least two equal sides -- we know that. It's also the case that an isosceles triangle must have at least two equal angles in it, namely the angles which are opposite the equal sides.

It's a fact from geometry that the sum of the three inside angles of any triangle is always  $180^\circ$ . For example, in the isosceles triangle above, the top angle must be  $86^\circ$ , because

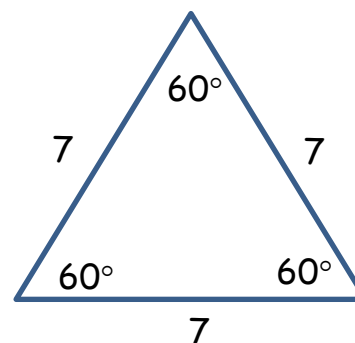
$$47^\circ + 47^\circ + 86^\circ = 180^\circ$$

Because an equilateral triangle has three equal sides, all the angles must be the same -- namely  $60^\circ$  each -- since

$$60^\circ + 60^\circ + 60^\circ = 180^\circ$$



An Isosceles Triangle



An Equilateral Triangle  
(It's also isosceles)

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## Homework

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1. Two of the angles in a triangle are  $100^\circ$  and  $30^\circ$ . What is the measure of the third angle?
2. Two of the angles in an isosceles triangle are  $35^\circ$  and  $35^\circ$ . Find the measure of the third angle.

3. Name the angles in any equilateral triangle.
4. True/False:
  - a. The two acute angles (the ones that are not  $90^\circ$ ) in a right triangle must be the same.
  - b. The two acute angles in a right triangle have a sum of  $90^\circ$ .

### □ Angle Problems

The previous chapter's triangle problems dealt with perimeter. The following problems deal with the three angles in a triangle. Notice that the problems do not specify what the sum of the three angles is -- that's because we know that **the sum of the inside angles in any triangle is  $180^\circ$**  (and it's your job to remember that fact!).

**EXAMPLE 1:      The second angle of a triangle is  $12^\circ$  less than 4 times the first angle. The third angle is  $30^\circ$  less than the second angle. What is the measure of each angle?**

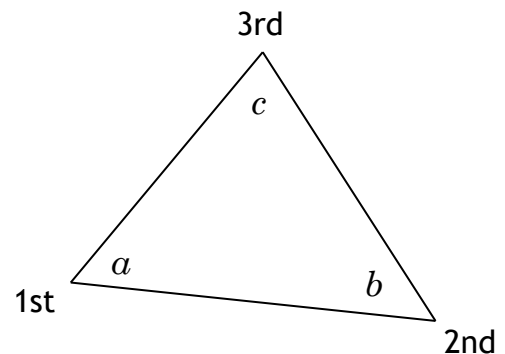
**Solution:** First, since the problem talks about the three angles of a triangle, the key to the equation will be that the angles must total  $180^\circ$ . We start by giving names to the three angles inside the triangle:

$$a = \text{1st angle}$$

$$b = \text{2nd angle}$$

$$c = \text{3rd angle}$$

Now we parse the given problem phrase by phrase:



The second angle of a triangle is  
 $12^\circ$  less than 4 times the first angle translates to

$$b = 4a - 12$$

The third angle is  $30^\circ$  less than the second angle becomes

$$c = b - 30$$

Our equation is

$$\begin{aligned}
 a + b + c &= 180 && \text{(angles total } 180^\circ\text{)} \\
 \Rightarrow a + \boxed{4a - 12} + \boxed{b - 30} &= 180 && \text{(substitute for } b \text{ \& } c\text{)} \\
 \Rightarrow 5a - 42 + b &= 180 && \text{(combine like terms)} \\
 \Rightarrow 5a - 42 + \boxed{4a - 12} &= 180 && \text{(substitute for } b\text{)} \\
 \Rightarrow 9a - 54 &= 180 && \text{(combine like terms)} \\
 \Rightarrow 9a &= 234 && \text{(add 54 to each side)} \\
 \Rightarrow a &= \mathbf{26} && \text{(divide by 9)}
 \end{aligned}$$

We have our final answers:

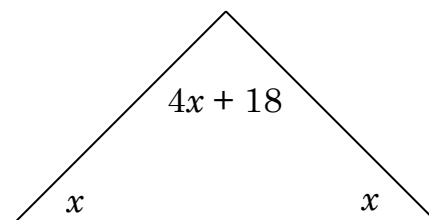
The first angle is  $a = \boxed{26^\circ}$

The second angle =  $b = 4a - 12 = 4(\mathbf{26}) - 12 = \boxed{92^\circ}$

The third angle =  $c = b - 30 = \mathbf{92} - 30 = \boxed{62^\circ}$

**EXAMPLE 2:** The third angle of an isosceles triangle is  $18^\circ$  more than 4 times one of the equal angles. What is the measure of each angle?

**Solution:** For variety, let's try solving this problem using just one variable. Since two of the angles must be equal,



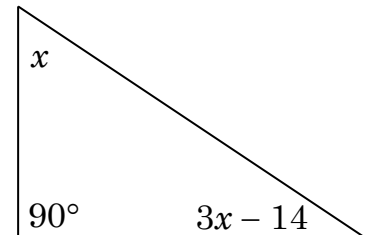
let's call each of them  $x$ . So the third angle, which is  $18^\circ$  more than 4 times  $x$ , can be written  $4x + 18$ . The sum of the three angles must be  $180^\circ$ :

$$\begin{aligned} x + x + (4x + 18) &= 180 && \text{(3 angles total } 180^\circ\text{)} \\ \Rightarrow x + x + 4x + 18 &= 180 && \text{(remove parentheses)} \\ \Rightarrow 6x + 18 &= 180 && \text{(combine like terms)} \\ \Rightarrow 6x &= 162 && \text{(subtract 18 from each side)} \\ \Rightarrow x &= \mathbf{27} && \text{(divide each side by 6)} \\ \Rightarrow 4x + 18 &= 4(\mathbf{27}) + 18 = 126 \end{aligned}$$

Hence, the three angles are  $27^\circ$ ,  $27^\circ$ , and  $126^\circ$

**EXAMPLE 3:** One of the acute angles of a right triangle is  $14^\circ$  less than 3 times the other acute angle. Find the measure of the larger acute angle.

**Solution:** Again, let's use just one variable to solve this problem. Since one of the angles in a right triangle is  $90^\circ$ , we need only concern ourselves with the two acute angles. Calling one of them  $x$  means that the other one must be called  $3x - 14$ . Since the sum of the three angles must be  $180^\circ$ , we get the equation



$$\begin{aligned} x + (3x - 14) + 90 &= 180 && \text{(3 angles total } 180^\circ\text{)} \\ \Rightarrow x + 3x - 14 + 90 &= 180 && \text{(drop the parentheses)} \\ \Rightarrow 4x + 76 &= 180 && \text{(combine like terms)} \\ \Rightarrow 4x &= 104 && \text{(subtract 76 from each side)} \\ \Rightarrow x &= \mathbf{26} && \text{(divide each side by 4)} \\ \Rightarrow 3x - 14 &= 3(\mathbf{26}) - 14 = 64 \end{aligned}$$

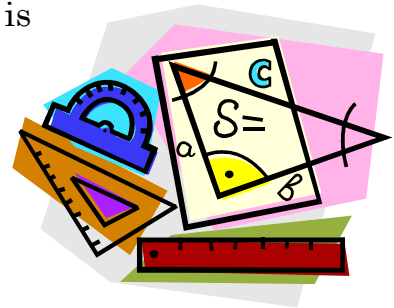
Therefore, the larger acute angle is  $64^\circ$

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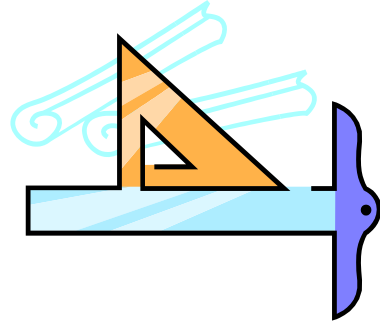
## Homework

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5. One of the acute angles of a right triangle is  $18^\circ$  more than 2 times the other acute angle. Find the measures of the two acute angles.
6. The third angle of an isosceles triangle is  $15^\circ$  more than 3 times one of the equal angles. What is the measure of each angle?
7. The second angle of a triangle is  $16^\circ$  more than 3 times the first angle. The third angle is  $125^\circ$  less than the second angle. What is the measure of each angle?
8. The third angle of an isosceles triangle is  $4^\circ$  less than 2 times one of the equal angles. What is the measure of each angle?
9. The second angle of a triangle is  $38^\circ$  less than 4 times the first angle. The third angle is  $13^\circ$  more than the second angle. What is the measure of each angle?
10. One of the acute angles of a right triangle is  $6^\circ$  less than 5 times the other acute angle. Find the measures of the two acute angles.
11. One of the acute angles of a right triangle is  $10^\circ$  less than 3 times the other acute angle. Find the measures of the two acute angles.
12. The third angle of an isosceles triangle is  $8^\circ$  less than 2 times one of the equal angles. What is the measure of each angle?
13. The second angle of a given triangle is  $66^\circ$  more than its first angle, while its third angle is  $18^\circ$  less than 4 times the first angle. Find the measure of each angle.



14. One of the acute angles of a right triangle is  $18^\circ$  less than 2 times the other acute angle. Find the measures of the two acute angles.
15. One of the acute angles of a right triangle is  $60^\circ$  less than 2 times the other acute angle. Find the measures of the two acute angles.
16. The third angle of an isosceles triangle is  $68^\circ$  less than 2 times one of the equal angles. What is the measure of each angle?
17. The second angle of a triangle is  $20^\circ$  less than the first angle. The third angle is  $20^\circ$  more than twice the second angle. What is the measure of each angle?



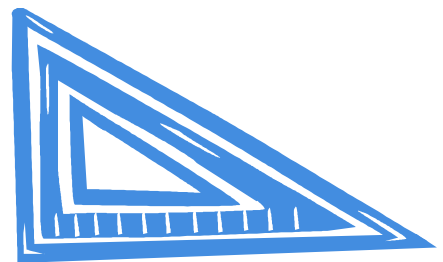
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## Review Problems

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18. The second angle of a triangle is  $30^\circ$  more than the first angle, while the first angle is  $26^\circ$  more than 3 times the third angle. What is the measure of each angle?
19. One of the acute angles of a right triangle is  $18^\circ$  more than 5 times the other acute angle. What is the measure of each acute angle?
20. The first angle of a triangle is  $1^\circ$  more than 8 times the third angle, while the second angle is  $47^\circ$  more than 3 times the third angle. What is the measure of each angle?
21. The third angle of a triangle is  $90^\circ$  less than 10 times the second angle, while the first angle is  $50^\circ$  less than 9 times the second angle. What is the measure of each angle?
22. One of the acute angles of a right triangle is  $10^\circ$  less than the other acute angle. What is the measure of each acute angle?

23. The second angle of a triangle is  $96^\circ$  less than 3 times the third angle, while the first angle is  $25^\circ$  less than 3 times the third angle. What is the measure of each angle?
24. The third angle of a triangle is  $38^\circ$  more than 3 times the second angle, while the first angle is  $43^\circ$  less than the third angle. What is the measure of each angle?
25. The second angle of a triangle is  $11^\circ$  more than 2 times the third angle, while the first angle is  $55^\circ$  less than 5 times the third angle. What is the measure of each angle?
26. One of the acute angles of a right triangle is  $14^\circ$  less than the other acute angle. What is the measure of each acute angle?
27. The second angle of a triangle is  $3^\circ$  less than 4 times the first angle, while the third angle is  $42^\circ$  more than the second angle. What is the measure of each angle?
28. The second angle of a triangle is  $32^\circ$  less than 6 times the first angle, while the third angle is  $42^\circ$  less than the second angle. What is the measure of each angle?
29. The third angle of an isosceles triangle is  $12^\circ$  more than 5 times one of the equal angles. What is the measure of each angle?
30. The second angle of a triangle is  $28^\circ$  more than 5 times the first angle, while the third angle is  $42^\circ$  more than 4 times the first angle. What is the measure of each angle?
31. The second angle of a triangle is  $75^\circ$  more than 5 times the first angle, while the third angle is  $21^\circ$  less than 3 times the first angle. What is the measure of each angle?
32. The third angle of an isosceles triangle is  $4^\circ$  more than 20 times one of the equal angles. What is the measure of each angle?
33. The second angle of a triangle is  $29^\circ$  more than the first angle, while the first angle is  $15^\circ$  more than 5 times the third angle. What is the measure of each angle?



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## Solutions

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|-------------------------------------|-------------------------------------|-------------------------------------|
| 1. $50^\circ$                       | 2. $110^\circ$                      | 3. $60^\circ, 60^\circ, 60^\circ$   |
| 4. a. F b. T                        | 5. $24^\circ, 66^\circ$             | 6. $33^\circ, 33^\circ, 114^\circ$  |
| 7. $39^\circ, 133^\circ, 8^\circ$   | 8. $46^\circ, 46^\circ, 88^\circ$   | 9. $27^\circ, 70^\circ, 83^\circ$   |
| 10. $16^\circ, 74^\circ$            | 11. $25^\circ, 65^\circ$            | 12. $47^\circ, 47^\circ, 86^\circ$  |
| 13. $22^\circ, 70^\circ, 88^\circ$  | 14. $36^\circ, 54^\circ$            | 15. $40^\circ, 50^\circ$            |
| 16. $56^\circ, 62^\circ, 62^\circ$  | 17. $35^\circ, 55^\circ, 90^\circ$  | 18. $14^\circ, 68^\circ, 98^\circ$  |
| 19. $12^\circ, 78^\circ$            | 20. $11^\circ, 89^\circ, 80^\circ$  | 21. $16^\circ, 70^\circ, 94^\circ$  |
| 22. $50^\circ, 40^\circ$            | 23. $43^\circ, 33^\circ, 104^\circ$ | 24. $21^\circ, 101^\circ, 58^\circ$ |
| 25. $28^\circ, 67^\circ, 85^\circ$  | 26. $52^\circ, 38^\circ$            | 27. $16^\circ, 61^\circ, 103^\circ$ |
| 28. $22^\circ, 100^\circ, 58^\circ$ | 29. $24^\circ, 24^\circ, 132^\circ$ | 30. $11^\circ, 83^\circ, 86^\circ$  |
| 31. $14^\circ, 145^\circ, 21^\circ$ | 32. $8^\circ, 8^\circ, 164^\circ$   | 33. $11^\circ, 70^\circ, 99^\circ$  |

“The beginning of knowledge is the discovery of something we do not understand.”

Frank Herbert