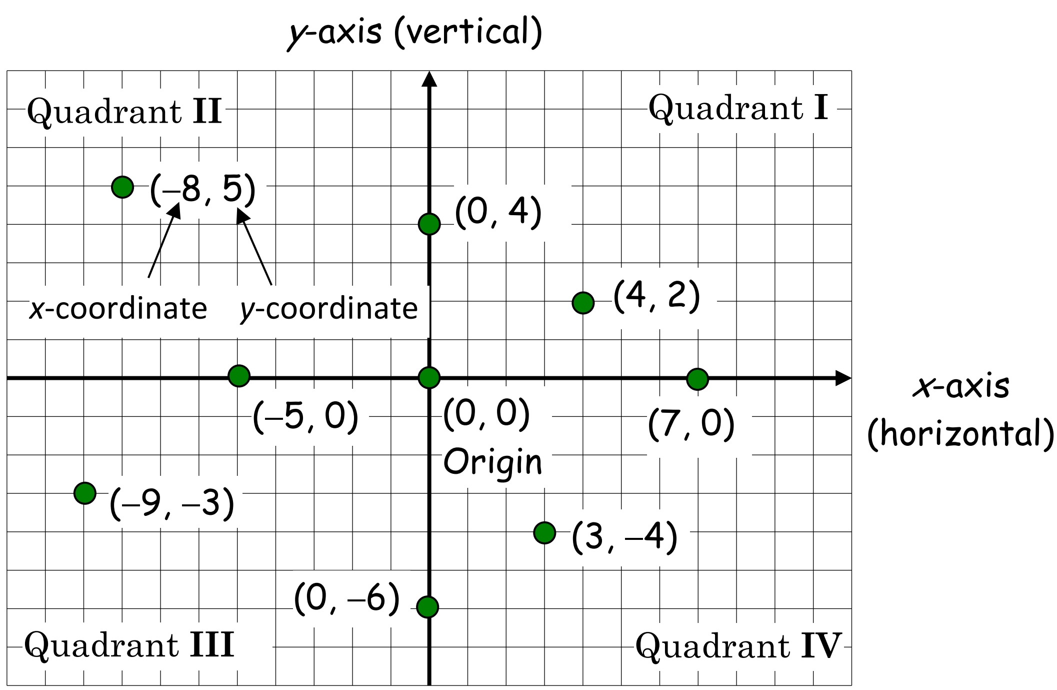
CH 35 – THE COORDINATE PLANE

❑ Introduction

Someone once said "A picture is worth a thousand words." This is especially true in math, where many ideas are very abstract. The French mathematician-philosopher René Descartes ("I think, therefore I am") devised a way for us to visually represent the solutions of many kinds of equations. It's called the ***Cartesian Coordinate System***. The term *Cartesian* is the Latin form of the name Descartes.

We've seen one-dimensional number lines before. In this chapter we take two number lines (one called the *x*-axis and one called the *y*-axis), line them up perpendicular to each other (90° angle between them), and we have a two-dimensional system called **2-space**. It's a fancy form of the "Battleship" game, or the basic layout of a spreadsheet.

❑ The Cartesian Coordinate System (2-Space)



Observations on 2-Space

1. A two-dimensional coordinate system (like the previous figure) represents a ***plane***. The horizontal axis is called the ***x*-axis** in math, but will be called other things in other subjects. Similarly, the vertical ***y*-axis** will be called something else in other subjects.

2. The ***ordered pair* (*x*, *y*)** represents a single ***point*** in the plane. The numbers x and y in the ordered pair are the ***coordinates*** of the point. Notice that a single point (ordered pair) consists of two coordinates. The coordinates of a point on the Earth are called its longitude and latitude.

3. The point (0, 0), where the axes intersect (cross), is called the ***origin***.

4. The first coordinate of the point (x, y) represents the distance to the right or left from the origin. The second coordinate represents the distance up or down. For example, the point $(3, -4)$ is plotted by starting at the origin, moving 3 units to the right, and then moving 4 units down.

5. The ***quadrants*** are numbered I (one) through IV (four), starting in the upper-right region and going counterclockwise.

In Quadrant I, both coordinates (the x and y) are positive.
In Quadrant II, x is negative and y is positive.
In Quadrant III, both coordinates are negative.
In Quadrant IV, x is positive and y is negative.

6. Points on the x-axis or the y-axis are not in any quadrant.

7. Every point on the x-axis has a y-coordinate of 0.
Every point on the y-axis has an x-coordinate of 0.

Homework

1. Is the Cartesian coordinate system described in this chapter 1-, 2-, or 3-dimensional?

2. a. Does the notation (x, y) represent one point or two points?

   b. Does the notation (x, y) represent one coordinate or two coordinates?

3. a. The point $(7, -\sqrt{3})$ lies in Quadrant ____.

   b. The point $(-\pi, -9)$ lies in Quadrant ____.

   c. The point $(-1, \sqrt{2})$ lies in Quadrant ____.

   d. The point $(\pi, \sqrt{5})$ lies in Quadrant ____.

4. a. The point (17, 0) lies on the ___ axis.

   b. The point (0, –20) lies on the ___ axis.

   c. The point (0, 0) is called the ____ and lies on the ___ axis.

5. a. In Quadrant I, x is _____ and y is _____.

   b. In Quadrant II, x is _____ and y is _____.

   c. In Quadrant III, x is _____ and y is _____.

   d. In Quadrant IV, x is _____ and y is _____.

6. a. In which quadrants are the signs of x and y the same?

   b. In which quadrants are the signs of x and y opposites?

7. a. A point lies on the x-axis. What can you say for sure about the coordinates of that point? Hint: The following are points on the x-axis: (7, 0), (–23, 0) and (π, 0).

   b. A point lies on the y-axis. What can you say for sure about the coordinates of that point?

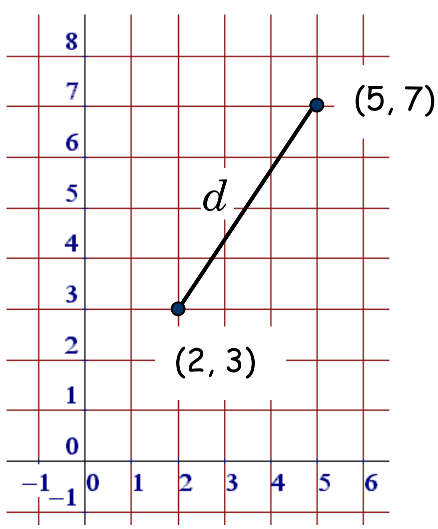
   c. A point lies on both axes. What can you say for sure about the coordinates of that point?

❑ The Distance Between Two Points

Now that we know how to plot points in the plane, how do we find the **distance** between two points in the plane? If the Earth were flat, it would be like asking how far apart two cities are if we know the latitude and longitude of each city.

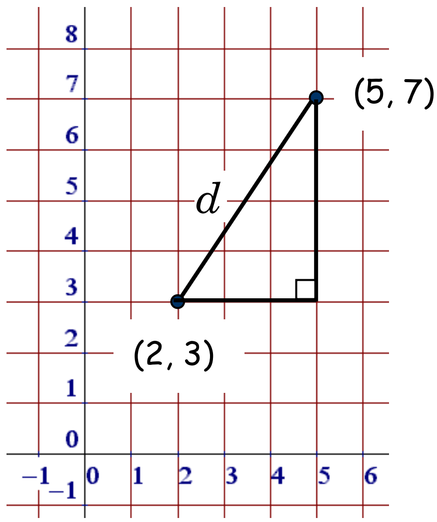
<u>EXAMPLE 1</u>: **Find the distance between the points (2, 3) and (5, 7) in the plane.**

<u>Solution</u>: Let's draw a picture and see what we can see. We'll plot the two given points and connect them with a straight line segment. The distance between the two points, which we'll call d, is simply the length of that line segment.



How far is it between the two points (2, 3) and (5, 7)? Equivalently, what is the length of the line segment connecting the two points?

Now what do we do? Well, here comes the interesting part. If we're creative enough, we might see that the segment connecting the two points can be thought of as the *hypotenuse* of a right triangle -- as long as we sketch in a pair of legs to create such a triangle. Let's do that:



We've created a right triangle whose legs have lengths 3 and 4, and whose hypotenuse has a length equal to the distance between the two given points.

Sure enough, we've constructed a right triangle where d is the length of the hypotenuse. If we can determine the lengths of the legs, then we can use the Pythagorean Theorem to find the length of the hypotenuse. By counting squares along the base of the triangle, we see that one leg is 3. Similarly, the other leg (the height) is 4. Since the square of the hypotenuse is equal to the sum of the squares of the legs, we can write the equation

$$
\begin{array}{llll}
 & d^2 = 3^2 + 4^2 & & \text{(Pythagorean Theorem: } \text{hyp}^2 = \text{leg}^2 + \text{leg}^2\text{)} \\
\Rightarrow & d^2 = 9 + 16 & & \text{(square the legs)} \\
\Rightarrow & d^2 = 25 & & \text{(add)} \\
\Rightarrow & d = 5 & & \text{(since } \sqrt{25} = 5\text{)}
\end{array}
$$

You may notice that $d = -5$ also satisfies the equation $d^2 = 25$, since $(-5)^2 = 25$. But does a negative value of d make sense? No, because distance can never be negative; so we conclude that

$$\boxed{\text{The distance between the two points is 5}}$$

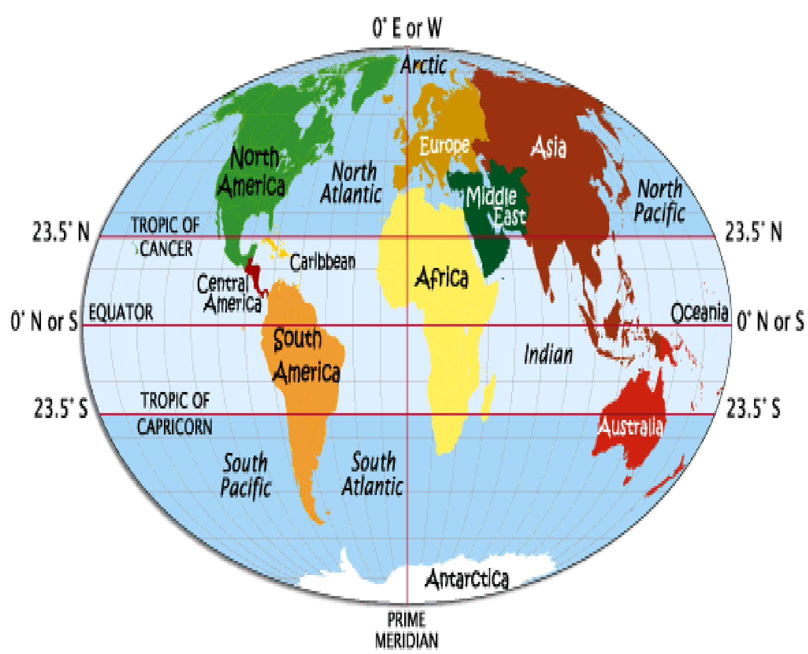
Homework

8. By plotting the two given points in the plane and using the Pythagorean Theorem, find the **distance** between the points:

a. (1, 1) and (4, 5)
b. (2, –3) and (6, –6)
c. (–3, 5) and (2, –7)
d. (–4, –5) and (1, 7)
e. (–5, 0) and (1, 8)
f. the origin and (6, 8)
g. the origin and (–5, –12)
h. (2, 5) and (2, –1)
i. (–3, 4) and (2, 4)
j. (π, 99) and (π, 99)

❑ Geography

One last connection between the *x*-*y* coordinate system and the longitude/latitude system used on the Earth: The *x*-axis can be thought of as the *Equator* (0° latitude), and the *y*-axis can be viewed as the *Prime Meridian* (0° longitude), a line of longitude going from the North Pole to the South Pole and passing through Greenwich, England, just outside London.



Review Problems

9. a. The y-axis is the (horizontal, vertical) axis.
   b. In the point (7, 9), the x-coordinate is _____.
   c. The point (0, 0) is called the _____.
   d. In which quadrant is x positive and y negative?
   e. Every point on the y-axis has an x-coordinate of _____.
   f. Which quadrant is the point $(-\pi, \sqrt{17})$ in?

10. Find the distance between the given pair of points:

    a. (0, 0) and (–5, 12) b. (–6, 8) and the origin

    c. (–1, –1) and (2, 3) d. (–3, 4) and (2, –8)

Solutions

1. The Cartesian coordinate system in this course is 2-dimensional. In later courses you will encounter a 3-dimensional coordinate system, and if you major in math or physics you will find even higher-dimension coordinate systems.

2. a. one b. two

3. a. IV b. III c. II d. I

4. a. x b. y c. origin; it's on <u>both</u> axes.

5. a. positive; positive b. negative; positive
c. negative; negative d. positive, negative

6. a. I and III b. II and IV

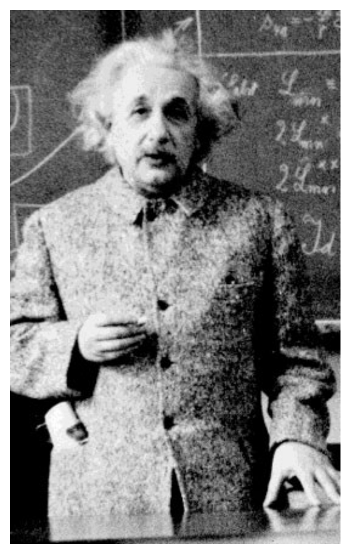
7. a. The y-coordinate must be 0.
b. The x-coordinate must be 0.
c. Both coordinates must be 0, since it's the origin.

8. a. 5 b. 5 c. 13 d. 13 e. 10
f. 10 g. 13 h. 6 i. 5 j. 0

9. a. vertical b. 7 c. origin
d. IV e. 0 f. II

10. a. 13 b. 10 c. 5 d. 13

"Where the world ceases to be the scene of our personal hopes and wishes, where we face it as free beings admiring, asking and observing, there we enter the realm of Art and Science."



Albert Einstein
(1879-1955)