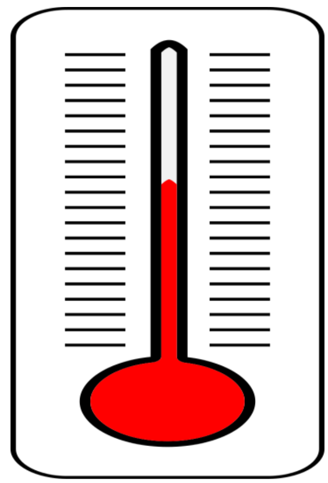
CH 42 – TEMPERATURE FORMULAS AND MORE

❑ Two Temperature Scales



On the Fahrenheit temperature scale, water freezes at 32°F and boils at 212°F. Later, the Celsius (originally called centigrade) scale was created to make the numbers easier to work with. On this new scale, water freezes at 0°C and boils at 100°C. We conclude that 0°C = 32°F and 100°C = 212°F.

❑ Creating the Formula $F = 1.8C + 32$

Our goal here is to create, from scratch, a formula which converts from degrees Celsius to degrees Fahrenheit. All we need to accomplish this goal is two facts:

0°C = 32°F

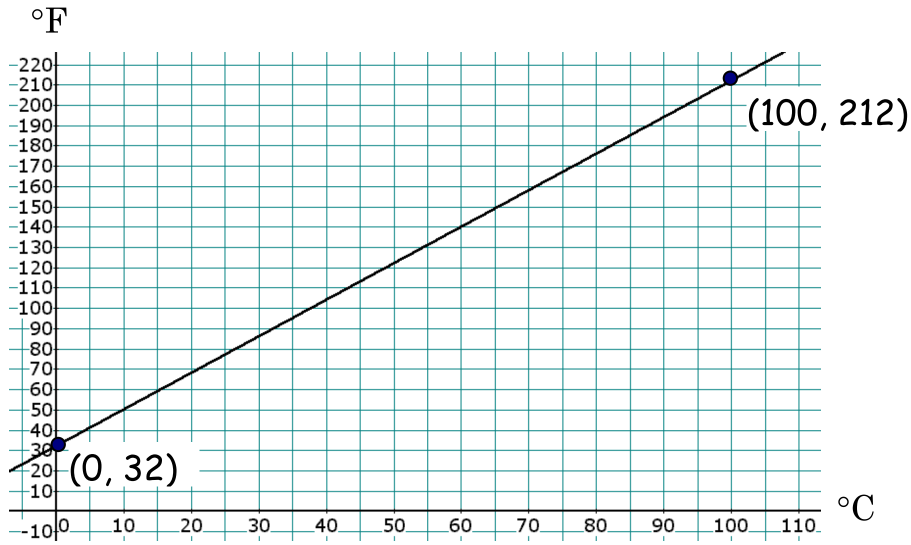
100°C = 212°F

and one assumption:

The relationship is linear (the graph is a straight line).

If we label the horizontal axis °C and the vertical axis °F, then the temperature facts above translate into the following two points on the line: (0, 32) and (100, 212).

As we proceed through the derivation, notice that °C is just a new name for x, and °F is a new name for y.



We begin with a linear formula relating °C to °F, where m represents the slope of the line, and b is the "F-intercept."

$$F = mC + b$$ (note that C is the x-value and F is the y-value)

First, we use the two points on the line to calculate the **slope** of the line:

$$m = \frac{\Delta F}{\Delta C} = \frac{212 - 32}{100 - 0} = \frac{180}{100} = 1.8$$

So now our equation is

$$F = 1.8C + b$$

Second, we use one of the given points -- we'll use (0, 32) -- to find b:

$$\mathbf{32} = 1.8(\mathbf{0}) + b \Rightarrow 32 = 0 + b \Rightarrow 32 = b$$

and we have our equation relating the two temperature scales:

$$\boxed{F = 1.8C + 32}$$

❑ Creating the Formula to Convert Fahrenheit to Celsius

What about the other way around?

> *What if we're given the Fahrenheit temperature and wish to calculate the Celsius temperature?*

There are three ways we can answer this question.

Answer #1: Use the formula we derived above, $F = 1.8C + 32$, plug in the given Fahrenheit temperature, and finish it off with a little algebra. For example, to convert 50°F to Celsius, we can write

$F = 1.8C + 32$ (the formula we derived)

$\mathbf{50} = 1.8C + 32$ (substitute 50 for F)

$$\begin{aligned} 50 &= 1.8C + 32 \\ \mathbf{-32} & \quad\;\; \mathbf{-\,32} \end{aligned}$$ (subtract 32 from each side)

$18 = 1.8C$ (simplify each side)

$10 = C$ (divide each side by 1.8)

Conclusion: <u>50°F is equivalent to 10°C</u>.

Answer #2: Just take the formula $F = 1.8C + 32$ and solve for C. After all, why not take advantage of our knowledge of solving formulas?

$F = 1.8C + 32$ (the formula to find F)

$\Rightarrow \quad F - 32 = 1.8C$ (subtract 32 from each side)

$\Rightarrow \quad \dfrac{F-32}{\mathbf{1.8}} = \dfrac{1.8C}{\mathbf{1.8}}$ (divide each side by 1.8)

$\Rightarrow \quad \dfrac{F-32}{1.8} = C$ (simpify)

And so we now have our Celsius formula: $\boxed{C = \dfrac{F-32}{1.8}}$

So to solve the problem from Answer #1 -- to convert 50°F to °C -- we just plug in 50 for F and calculate:

$$C = \frac{F-32}{1.8} = \frac{\mathbf{50}-32}{1.8} = \frac{18}{1.8} = 10$$

and again we conclude that 50°F is equivalent to 10°C.

Answer #3: We could derive the formula for °C by mimicking the derivation of the formula $F = 1.8C + 32$ at the beginning of this chapter.

Homework

1. Derive the temperature formula $C = \frac{F-32}{1.8}$ in the same manner that the formula $F = 1.8C + 32$ was derived.

2. On day 5 the height of a plant is measured to be 38.1 cm. On day 9 the height has increased to 63.7 cm.

   a. Find the linear formula that expresses the height h of plant as a function of the day d.

   b. Use your formula to predict the height of the tree on day 11.

   c. On what day is the plant 127.7 cm tall?

   d. What does the 6.4 in your formula represent?

   e. Solve the formula for d.

3. During a recent snowstorm, there were 12 inches of snow on the ground at 7 p.m., and there were 19 inches at 11 p.m. If you were to graph this data (with time on the horizontal axis and snow on the vertical axis), calculate the slope, and then describe what the slope of the line represents in the context of this problem.

4. A scientist measured the pressure at a depth of 2 ft and determined that it was 15.59 lbs/in^2 (pounds per square inch). At a depth of 10 ft the pressure was 19.15 lbs/in^2.

a. Find the linear formula that expresses the pressure P as a function of the depth d. [Round to 3 digits.]

b. Use your formula to predict the pressure at a depth of 100 ft.

c. At what depth would we find a pressure of 85.9 lbs/in^2?

d. What does the 0.445 in your formula represent?

e. According to the formula, what would be the pressure at sea level ($d = 0$)? Does this make any sense?

f. Solve the formula for d.

Solutions

1. Interchange the °C and °F axes, switch the coordinates of the two points on the graph, and proceed with the ideas of slope and intercept.

2. a. $h = 6.4d + 6.1$ b. 76.5 cm c. day 19

d. It's the amount the plant grows each day (on average)

e. $d = \dfrac{h-1}{6.4}$

3. $m = \dfrac{7}{4}$ in/hr

The slope represents the average hourly rate of change in the snow level. That is, it's a measure of how fast it's snowing.

4. a. $P = 0.445d + 14.7$ b. 59.2 lbs/in^2 c. 160 ft

d. The increase in pressure for each additional foot below sea level.

e. The pressure would be 14.7 lbs/in2. It's not an error. Can you account for the pressure? Hint: It's not water pressure.

f. $d = \frac{P - 14.7}{0.445}$

"It does not matter how slowly you go, as long as you do not stop."

– Confucius –