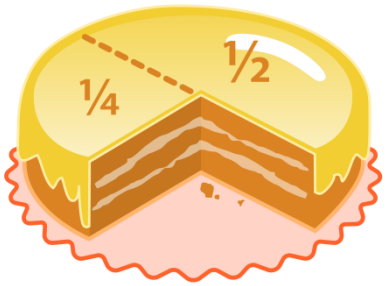
CH 48 – MORE FRACTIONS

❑ Introduction

In this chapter we tie up some loose ends. First, we split a single fraction into two fractions, followed by performing our standard math operations on positive and negative fractions. Then we solve equations and number problems which contain fractions.



❑ Splitting Up a Fraction

Before this course is over we will see the need to "split up" a fraction in a couple of ways, one using multiplication and the other using addition.

EXAMPLE 1:

A. **Express $\frac{7x}{2}$ as the product of two quantities:**

Solution: The easiest way to see this process is to just do it and then check that it's right. Here's the claim:

$$\frac{7x}{2} = \frac{7}{2}x$$

and here's the reason:

$$\frac{7}{2}x = \frac{7}{2}\cdot\frac{x}{1} = \frac{7\cdot x}{2\cdot 1} = \frac{7x}{2} \quad \checkmark$$

B. **Express $\frac{7x+9}{5}$ as the sum of two fractions.**

Solution: What two fractions have a sum of $\frac{7x+9}{5}$? The 5 tells us that we could use fractions each with a denominator of 5. Since the numerator is $7x + 9$, we can make one of the numerators $7x$ and the other one 9. Thus,

$$\frac{7x+9}{5} = \mathbf{\frac{7x}{5} + \frac{9}{5}}$$ (check by adding the two fractions)

C. **Combine the ideas of parts A and B to split up $\frac{8x-5}{7}$.**

Solution:

$$\frac{8x-5}{7} = \frac{8x}{7} - \frac{5}{7} = \mathbf{\frac{8}{7}x - \frac{5}{7}}$$

D. **Express $\frac{8x+16}{16}$ as the sum of two quantities.**

Solution:

$$\frac{8x+16}{16} = \frac{8x}{16} + \frac{16}{16} = \frac{x}{2} + 1 = \mathbf{\frac{1}{2}x + 1}$$

Homework

1. Express each fraction as the product of two quantities, using part A of Example 1 as a guide:

a. $\frac{9x}{4}$ b. $\frac{17x}{2}$ c. $\frac{3y}{16}$ d. $\frac{-33a}{17}$ e. $\frac{5n}{-23}$

f. $\frac{3x}{6}$ g. $\frac{-6x}{-2}$ h. $\frac{22m}{33}$ i. $\frac{-9x}{-15}$ j. $\frac{-39z}{52}$

2. Express each fraction as the sum or difference of two quantities, using parts C and D of Example 1 as a guide:

a. $\frac{3x+8}{4}$ b. $\frac{-9x+18}{6}$ c. $\frac{y-1}{8}$ d. $\frac{3n+15}{-5}$

e. $\frac{-3w-24}{2}$ f. $\frac{45x-75}{15}$ g. $\frac{33x+44}{55}$ h. $\frac{-31x-17}{-20}$

❑ Signed Fractions

Adding and Subtracting

$$-\frac{2}{3}-\frac{1}{2} = -\frac{4}{6}-\frac{3}{6} = -\mathbf{\frac{7}{6}}$$

$$\frac{4}{5}-\left(-\frac{2}{3}\right) = \frac{4}{5}+\frac{2}{3} = \frac{12}{15}+\frac{10}{15} = \mathbf{\frac{22}{15}}$$

$$\frac{2}{9}-7 = \frac{2}{9}-\frac{63}{9} = -\mathbf{\frac{61}{9}}$$

Multiplying and Dividing

$$\left(\frac{2}{3}\right)\left(-\frac{5}{7}\right) = -\mathbf{\frac{10}{21}}$$

$$-\frac{4}{7}\div -2 = -\frac{4}{7}\times -\frac{1}{2} = \frac{4}{14} = \mathbf{\frac{2}{7}}$$

$$\left(-\frac{1}{2}\right)\left(-\frac{1}{3}\right)\left(-\frac{1}{4}\right)\left(-\frac{1}{5}\right) = \mathbf{\frac{1}{120}}$$

$$\frac{-8}{\frac{1}{3}} = -\frac{8}{1}\times\frac{3}{1} = -\mathbf{24}$$

$$\frac{-\frac{9}{4}}{-2} = -\frac{9}{4}\div -2 = -\frac{9}{4}\times -\frac{1}{2} = \mathbf{\frac{9}{8}}$$

Powers and Square Roots

An **exponent** still means what it always has, so these next examples should be clear.

$$\left(\frac{2}{3}\right)^2 = \frac{2}{3}\cdot\frac{2}{3} = \mathbf{\frac{4}{9}}$$

$$\left(-\frac{1}{4}\right)^3 = \left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right) = \mathbf{-\frac{1}{64}}$$

$$\left(-\frac{9}{4}\right)^1 = \mathbf{-\frac{9}{4}}$$

$$\left(\frac{123}{456}\right)^0 = \mathbf{1}$$

$$\left(\frac{1}{2}\right)^8 = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \mathbf{\frac{1}{256}}$$

As for the **square root** sign, we still ask the question: What number (that's not negative) times itself gives the number in the radical sign?

$\sqrt{\frac{9}{25}} = \mathbf{\frac{3}{5}}$ This is true because $\left(\frac{3}{5}\right)^2 = \frac{9}{25}$.

$\sqrt{\frac{1}{144}} = \mathbf{\frac{1}{12}}$ This is because $\frac{1}{12}\times\frac{1}{12} = \frac{1}{144}$.

$\sqrt{-\frac{4}{49}}$ does <u>not</u> exist, because $-\frac{4}{49}$ is a negative number, and square roots of negative numbers don't exist in this course.

$\sqrt{\frac{-4}{-49}}$ does exist, because the fraction is actually a positive number: $\sqrt{\frac{-4}{-49}} = \sqrt{\frac{4}{49}} = \mathbf{\frac{2}{7}}$.

Homework

Perform the indicated operation:

3. a. $-\frac{1}{2}-\frac{4}{5}$ b. $-\frac{1}{3}-\left(-\frac{1}{3}\right)$ c. $\frac{2}{3}-\left(-\frac{5}{6}\right)$

   d. $-\frac{4}{5}+\frac{2}{3}$ e. $9-\frac{4}{5}$ f. $-1-\frac{2}{3}$

   g. $\frac{8}{3}-5$ h. $-\frac{2}{3}-(-1)$ i. $-\frac{1}{4}-\frac{2}{7}$

4. a. $\left(-\frac{1}{2}\right)\left(-\frac{5}{6}\right)$ b. $\left(-\frac{2}{3}\right)\left(\frac{3}{2}\right)$ c. $-\frac{5}{6}\cdot-\frac{6}{5}$

   d. $-\frac{2}{3}\div-\frac{3}{2}$ e. $\frac{1}{2}\div-9$ f. $7\div-\frac{3}{4}$

   g. $\dfrac{-\frac{2}{3}}{-\frac{1}{9}}$ h. $\dfrac{\frac{4}{5}}{-8}$ i. $\dfrac{-\frac{4}{5}}{\frac{5}{8}}$

5. a. $\left(-\frac{2}{3}\right)^2$ b. $\left(-\frac{1}{2}\right)^3$ c. $\left(-\frac{1}{3}\right)^4$

   d. $\left(-\frac{9}{10}\right)^0$ e. $\left(-\frac{14}{19}\right)^1$ f. $\left[\frac{1}{2}-\frac{9}{10}\right]^0$

   g. $\left(-\frac{1}{2}\right)^5$ h. $\left(-\frac{2}{3}\right)^6$ i. $\left(\frac{99}{-99}\right)^{99}$

6. a. $\sqrt{\frac{81}{100}}$ b. $\sqrt{\frac{36}{64}}$ c. $\sqrt{\frac{1}{4}}$

d. $\sqrt{\frac{1}{9}}$ e. $\sqrt{\frac{121}{144}}$ f. $\sqrt{-\frac{25}{81}}$

g. $\sqrt{\frac{-256}{-289}}$ h. $\sqrt{-\frac{14}{17}-\left(-\frac{14}{17}\right)}$ i. $\sqrt{(abc)^0}$

❑ Equations and Word Problems

EXAMPLE 2: **Solve for x:** $-\frac{\mathbf{2}}{\mathbf{3}}\boldsymbol{x}-\frac{\mathbf{1}}{\mathbf{8}}=\frac{\mathbf{5}}{\mathbf{6}}$

Solution: To isolate the x, we will first add $\frac{1}{8}$ to each side of the equation:

$$-\frac{2}{3}x-\frac{1}{8}+\mathbf{\frac{1}{8}}=\frac{5}{6}+\mathbf{\frac{1}{8}}$$

Now we simplify each side of the equation:

Left side: $-\frac{2}{3}x-\frac{1}{8}+\frac{1}{8}=-\frac{2}{3}x$

Right side: $\frac{5}{6}+\frac{1}{8}=\frac{5}{6}\left[\frac{4}{4}\right]+\frac{1}{8}\left[\frac{3}{3}\right]=\frac{20}{24}+\frac{3}{24}=\frac{23}{24}$

The LCD is 24

So now the equation reads

$$-\frac{2}{3}x=\frac{23}{24}$$

The last step is to divide each side of the equation by $-\frac{2}{3}$ (in order to isolate the x):

$$\frac{-\frac{2}{3}x}{-\mathbf{\frac{2}{3}}}=\frac{\frac{23}{24}}{-\mathbf{\frac{2}{3}}}$$

$$\Rightarrow \quad \frac{-\frac{2}{3}x}{-\frac{2}{3}} = \frac{23}{24}\left(-\frac{3}{2}\right) \qquad \text{(multiply by the reciprocal)}$$

$$\Rightarrow \quad x = \frac{23}{3 \cdot 8} \cdot -\frac{3}{2} \qquad \text{(factor and cross-cancel)}$$

$$\Rightarrow \quad \boxed{x = -\frac{23}{16}}$$

EXAMPLE 3: **I'm thinking of a number. If $\frac{1}{2}$ is added to 3 times the number, the result is $\frac{3}{4}$. Find the number.**

Solution: We've seen this type of word problem before. The only new issue is what to do with those fractions. Suppose we let n represent the unknown number. The equation which results from the wording of the problem is

$$3n + \frac{1}{2} = \frac{3}{4}$$

We could subtract $\frac{1}{2}$ from each side of the equation and then divide by 3, or we could try a different technique, and here it is.

Determine the LCD of all the fractions in the problem. Noting that the denominator of the first term $3n$ is 1, we calculate the LCD of 1, 2 and 4 to be 4.

Now multiply each side of the equation by the LCD, 4:

$$\mathbf{4}\left[3n + \frac{1}{2}\right] = \mathbf{4}\left[\frac{3}{4}\right]$$

Distribute:

$$4\left[3n\right] + 4\left[\frac{1}{2}\right] = 4\left[\frac{3}{4}\right]$$

And simplify:

$$12n + 2 = 3$$

Solving for n gives the result $n = \frac{1}{12}$

Therefore, the number I was thinking of is

$$\boxed{\frac{1}{12}}$$

Homework

7. Solve each equation: (You should solve some of them using the method of Example 3, and some with Example 4.)

a. $\frac{1}{2}x + 2 = -5$ b. $\frac{2}{3}x - \frac{1}{2} = \frac{1}{2}$ c. $\frac{2}{3}n + \frac{1}{2} = \frac{1}{3}$

d. $\frac{3}{4}t - \frac{1}{4} = \frac{1}{4}$ e. $-\frac{2}{3}n + \frac{1}{2} = -\frac{1}{2}$ f. $-\frac{1}{9}z - \frac{2}{3} = \frac{1}{3}$

g. $\frac{1}{2}x + \frac{3}{4} = -\frac{6}{8}$ h. $\frac{2}{5}a + \frac{1}{2} = -\frac{1}{10}$ i. $-\frac{2}{3}y - \frac{1}{4} = 12$

8. I'm thinking of a number. If $\frac{2}{3}$ is added to 4 times the number, the result is $\frac{9}{10}$. Find the number.

9. I'm thinking of a number. If $\frac{1}{2}$ is subtracted from 3 times the number, the result is –1. Find the number.

10. I'm thinking of a number. If $\frac{1}{7}$ is added to twice the number, the result is $-\frac{9}{10}$. Find the number.

11. I'm thinking of a number. If $\frac{2}{5}$ is subtracted from 5 times the number, the result is $-\frac{1}{2}$. Find the number.

12. If 5 times a number is increased by $\frac{2}{3}$, the result is –1. What is the number?

13. If –3 times a number is decreased by $\frac{3}{4}$, the result is $\frac{1}{2}$. What is the number?

14. If 4 times a number is increased by $\frac{1}{5}$, the result is $-\frac{2}{3}$. What is the number?

15. If –6 times a number is decreased by $\frac{2}{7}$, the result is 5. What is the number?

Review Problems

16. Express $\frac{-18y}{20}$ as a product.

17. Express $\frac{6x-11}{-2}$ as the sum of two quantities.

18. Evaluate each expression:

a. $-\frac{1}{2}+\frac{4}{5}$ b. $-\frac{1}{3}-\left(-\frac{1}{3}\right)$ c. $12-\frac{9}{13}$ d. $-\frac{2}{3}-\frac{1}{2}$

e. $\left(-\frac{1}{2}\right)\left(-\frac{1}{5}\right)$ f. $\frac{-\frac{2}{3}}{-\frac{1}{4}}$ g. $\left(-\frac{19}{40}\right)^0$ h. $\left(-\frac{1}{3}\right)^2$

i. $\left(-\frac{3}{4}\right)^3$ j. $\sqrt{\frac{9}{144}}$ k. $-\sqrt{\frac{25}{64}}$ l. $\sqrt{-\frac{9}{16}}$

19. Solve each equation:

a. $\frac{2}{3}x+\frac{1}{2}=\frac{1}{3}$ b. $-\frac{4}{5}y+\frac{1}{2}=-\frac{2}{3}$ c. $\frac{1}{4}z-\frac{3}{4}=-3$

20. I'm thinking of a number. If $\frac{1}{3}$ is added to 7 times the number, the result is $-\frac{1}{2}$. Find the number.

21. If 8 times a number is decreased by $\frac{2}{5}$, the result is –9. What is the number?

Solutions

1. a. $\frac{9}{4}x$ b. $\frac{17}{2}x$ c. $\frac{3}{16}y$ d. $-\frac{33}{17}a$
e. $-\frac{5}{23}n$ f. $\frac{1}{2}x$ g. $3x$ h. $\frac{2}{3}m$
i. $\frac{3}{5}x$ j. $-\frac{3}{4}z$

2. a. $\frac{3}{4}x+2$ b. $-\frac{3}{2}x+3$ c. $\frac{1}{8}y-\frac{1}{8}$ d. $-\frac{3}{5}n-3$
e. $-\frac{3}{2}w-12$ f. $3x-5$ g. $\frac{3}{5}x+\frac{4}{5}$ h. $\frac{31}{20}x+\frac{17}{20}$

3. a. $-\frac{13}{10}$ b. 0 c. $\frac{3}{2}$ d. $-\frac{2}{15}$ e. $\frac{41}{5}$
f. $-\frac{5}{3}$ g. $-\frac{7}{3}$ h. $\frac{1}{3}$ i. $-\frac{15}{28}$

4. a. $\frac{5}{12}$ b. -1 c. 1 d. $\frac{4}{9}$ e. $-\frac{1}{18}$
f. $-\frac{28}{3}$ g. 6 h. $-\frac{1}{10}$ i. $-\frac{32}{25}$

5. a. $\frac{4}{9}$ b. $-\frac{1}{8}$ c. $\frac{1}{81}$ d. 1 e. $-\frac{14}{19}$
f. 1 g. $-\frac{1}{32}$ h. $\frac{64}{729}$ i. -1

6. a. $\frac{9}{10}$ b. $\frac{3}{4}$ c. $\frac{1}{2}$ d. $\frac{1}{3}$ e. $\frac{11}{12}$
f. Does not exist g. $\frac{16}{17}$ h. 0 i. 1

7. a. -14 b. $\frac{3}{2}$ c. $-\frac{1}{4}$ d. $\frac{2}{3}$ e. $\frac{3}{2}$
f. -9 g. -3 h. $-\frac{3}{2}$ i. $-\frac{147}{8}$

8. $4n + \frac{2}{3} = \frac{9}{10}$

$\Rightarrow \quad 4n = \frac{9}{10} - \frac{2}{3} = \frac{27}{30} - \frac{20}{30} = \frac{7}{30}$

$\Rightarrow \quad 4n = \frac{7}{30} \Rightarrow n = \frac{\frac{7}{30}}{4} = \frac{7}{30} \div 4 = \frac{7}{30} \cdot \frac{1}{4} = \frac{7}{120}$

Alternate (and easier) Method: Multiply each side of the equation by the LCD, 30.

9. $3n - \frac{1}{2} = -1 \Rightarrow n = -\frac{1}{6}$ **10**. $-\frac{73}{140}$ **11**. $-\frac{1}{50}$

12. $-\frac{1}{3}$ **13**. $-\frac{5}{12}$ **14**. $-\frac{13}{60}$ **15**. $-\frac{37}{42}$

16. $-\frac{9}{10}y$ **17**. $-3x + \frac{11}{2}$

18. a. $\frac{3}{10}$ b. 0 c. $\frac{147}{13}$ d. $-\frac{7}{6}$ e. $\frac{1}{10}$

f. $\frac{8}{3}$ g. 1 h. $\frac{1}{9}$ i. $-\frac{27}{64}$ j. $\frac{1}{4}$

k. $-\frac{5}{8}$ l. Does not exist

19. a. $-\frac{1}{4}$ b. $\frac{35}{24}$ c. -9

20. $7n+\frac{1}{3}=-\frac{1}{2}$; $n=-\frac{5}{42}$

21. $8n-\frac{2}{5}=-9$; $n=-\frac{43}{40}$

"You can know the name of a bird in all the languages of the world, but when you're finished, you'll know absolutely nothing whatever about the bird... So let's look at the bird and see what it's doing -- that's what counts. I learned very early the difference between knowing the name of something and knowing something."

Richard Feynman