CH 51 – MORE EXPONENTS

❑ Introduction

This chapter is a continuation of the exponent ideas we've used many times before. Our goal is to combine expressions with exponents in them. First, a quick review of exponents:

$2^3 = 8$ $\quad -3^4 = -81$ $\quad 1^{124} = 1$ $\quad 0^{433} = 0$

$(-4)^3 = -64$ $\quad (-2)^6 = 64$ $\quad 2^0 = 1$ $\quad (8-6)^0 = 1$

❑ Review of Strange Exponents

We've worked with base 2 extensively, analyzing what happens when we apply various exponents to it. First we observed that $2^1 = 2$. It's reasonable to conclude that if x is any number,

$$\boxed{x^1 = x}$$

Next we encountered the exponent 0. 2^0 equaled 1, so we take a leap of faith and generalize that for any x (that's not zero),

$$\boxed{x^0 = 1}$$ [0^0 has no meaning until calculus]

Then came the strange negative exponent. We saw, for example, that $2^{-10} = \frac{1}{2^{10}}$. It's also the case that for any non-zero base x,

$$\boxed{x^{-n} = \frac{1}{x^n}} \quad \text{where } n = 1, 2, 3, \ldots$$

Zero raised to a negative power will be dealt with in the homework.

Homework

1. Evaluate each power:

a. 17^1	b. 999^1	c. $(3.8)^1$	d. $(-7)^1$
e. $\left(\frac{2}{5}\right)^1$	f. 19^0	g. $(-2.3)^0$	h. $\left(\frac{1}{4}\right)^0$
i. 3^{-1}	j. 4^{-2}	k. 5^{-3}	l. 6^{-4}

2. Find the value of the expression $\left(\dfrac{7x^3 - \sqrt{a-b} - \int_0^{\pi/2} \sin x \, dx}{\ln\left(\frac{1}{\tan\theta}\right)^{e^{i\pi}}} \right)^0$.

3. Assuming all variables are not zero, simplify each expression:

a. x^1	b. y^0	c. n^{-3}	d. z^{-5}
e. $(abc + 1)^0$	f. a^1	g. b^0	h. $(xy)^1$
i. t^{-1}	j. $(xy)^0 + 1$	k. $(xy)^0$	l. xy^0
m. $x^0 y$	n. $(x + y)^0$	o. $x + y^0$	p. $a^0 - b$
q. $(c - d)^0$	r. $10 \cdot 2^0$	s. w^{-2}	t. z^{-1}
u. a^{-5}	v. u^{-8}	w. n^{-4}	x. w^{-6}
y. a^{-7}	z. k^{-8}		

Hint: Exactly what is being raised to the 0 power?

4. Prove that 0^{-2} is undefined.

❑ Combining Things with Exponents

EXAMPLE 1: **Simplify each exponential expression using the stretch-and-squish technique.**

Stretch-and-squish means to expand the bases in the problem using the definition of exponents, then do some kind of simplifying using previous techniques, and then squish it back to exponent form.

A. $\boldsymbol{n^3 \times n^5}$

$$n^3 \times n^5 = (n \cdot n \cdot n) \times (n \cdot n \cdot n \cdot n \cdot n) = nnnnnnnn = \boldsymbol{n^8}$$

B. $\boldsymbol{\frac{x^6}{x^2}}$

$$\frac{x^6}{x^2} = \frac{xxxxxx}{xx} = \frac{\cancel{xx}xxxx}{\cancel{xx}} = \boldsymbol{x^4}$$

C. $\boldsymbol{\frac{n^3}{n^7}}$

$$\frac{n^3}{n^7} = \frac{nnn}{nnnnnnn} = \frac{\cancel{nnn}}{\cancel{nnn}nnnn} = \frac{1}{nnnn} = \boldsymbol{\frac{1}{n^4}}$$

D. $\boldsymbol{(ab)^3}$

	$(ab)^3$	
=	$(ab)(ab)(ab)$	(definition of cubing)
=	$ababab$	(parentheses not necessary)
=	$(aaa)(bbb)$	(rearrange the factors)
=	$\boldsymbol{a^3b^3}$	(rewrite with exponents)

E. $\boldsymbol{(uwz)^2}$

$$(uwz)^2 = (uwz)(uwz) = (uu)(ww)(zz) = \boldsymbol{u^2w^2z^2}$$

F. $\boldsymbol{\left(\frac{x}{y}\right)^2}$

$$\left(\frac{x}{y}\right)^2 = \left(\frac{x}{y}\right)\left(\frac{x}{y}\right) = \frac{xx}{yy} = \boldsymbol{\frac{x^2}{y^2}}$$

G. $\boldsymbol{\left(\frac{a}{b}\right)^3}$

$$\left(\frac{a}{b}\right)^3 = \left(\frac{a}{b}\right)\left(\frac{a}{b}\right)\left(\frac{a}{b}\right) = \frac{aaa}{bbb} = \boldsymbol{\frac{a^3}{b^3}}$$

H. $\boldsymbol{(x^4)^2}$

$$(x^4)^2 = (x^4)(x^4) = (xxxx)(xxxx) = xxxxxxxx = \boldsymbol{x^8}$$

EXAMPLE 2: **Simplify each expression:**

A. $x^2x^5 = (xx)(xxxxx) = xxxxxxx = \boldsymbol{x^7}$

B. $n^3 + n^7 = nnn + nnnnnnn = ???$

Note: We don't have a simple product of n's (due to the plus sign), so we can't write the sum using a single exponent. Besides, n^3 and n^7 are unlike terms, and therefore cannot be added. However we look at it, this problem cannot be simplified. On a test you can write your answer either as $\boldsymbol{n^3 + n^7}$ or **As is**.

C. $a^4 + a^4 = \boldsymbol{2a^4}$ These are like terms, so they add up.

D. u^3w^7 cannot be simplified, since $(uuu)(wwwwwww)$ is just what it is, 3 factors of u multiplied by 7 factors of w.

E. $y^3y^4 - y^7 = (yyy)(yyyy) - y^7 = y^7 - y^7 = \mathbf{0}$

F. $(-2c^3)(-10c^5) = (-2)(-10)(ccc)(ccccc) = 20(cccccccc) = \mathbf{20c^8}$

G. $(aqt)^3 = (aqt)(aqt)(aqt) = (aaa)(qqq)(ttt) = \mathbf{a^3q^3t^3}$

H. $3(a^3b^4)^2$

$= 3(a^3b^4)(a^3b^4)$ (the 3 is not being squared)

$= 3(aaa)(bbbb)(aaa)(bbbb)$ (stretch)

$= 3(aaaaaa)(bbbbbbbb)$ (rearrange factors)

$= \mathbf{3a^6b^8}$ (squish)

I. $(-2xy^2)^4$

$= (-2xy^2)\,(-2xy^2)\,(-2xy^2)\,(-2xy^2)$ (stretch)

$= (-2)(-2)(-2)(-2)(x)(x)(x)(x)(yy)(yy)(yy)(yy)$ (stretch)

$= 16(xxxx)(yyyyyyyy)$ (rearrange)

$= \mathbf{16x^4y^8}$ (squish)

J. $\left(\dfrac{g^4}{h^3}\right)^2$

$= \left(\dfrac{g^4}{h^3}\right)\left(\dfrac{g^4}{h^3}\right) = \dfrac{g^4 \cdot g^4}{h^3 \cdot h^3} = \dfrac{gggg \cdot gggg}{hhh \cdot hhh} = \mathbf{\dfrac{g^8}{h^6}}$

Homework

Use the stretch-and-squish technique (where appropriate) to simplify each expression:

5. a. $n^3 n^3$ b. xx^4 c. $z^4 z^4$ d. $a^4 a^5 a^3$
   e. $u^3 + u^4$ f. $w^9 + w^9$ g. $v^4 - v^3$ h. $c^{12} - c^{12}$

6. a. $\frac{x^8}{x^2}$ b. $\frac{y^9}{y^9}$ c. $\frac{z^7}{z^8}$ d. $\frac{a^7}{b^4}$

7. a. $(uv)^3$ b. $(abc)^2$ c. $(xy)^4$
   d. $(mnpq)^2$ e. $(jk)^2$ f. $(axy)^1$
   g. $(ax + b)^0$

8. True/False: $(x + y)^2 = x^2 + y^2$
   Check it out by using numbers for x and y.

9. a. $\left(\frac{a}{b}\right)^2$ b. $\left(\frac{w}{z}\right)^3$ c. $\left(\frac{x}{y}\right)^4$ d. $\left(\frac{1}{n}\right)^5$
   e. $\left(\frac{u}{v}\right)^1$ f. $\left(\frac{f}{g}\right)^0$ g. $\left(\frac{x}{2}\right)^5$ h. $(a - 5)^2$

10. a. $(2^3)^2$ b. $(3^2)^3$ c. $(1^4)^3$ d. $(0^2)^{44}$
    e. $(x^4)^2$ f. $(a^0)^4$ g. $(n^{33})^0$ h. $(m^0)^0$

11. a. $(2x)(3x)$ b. $(3a^2)(-2a)$ c. $5y^3(2y^3)$
    d. $(-x^2)(-x^4)$ e. $a^5 + a^3$ f. $9n^5 + 8n^5$
    g. $10q^3 - 10q^3$ h. $(2xy)(-3y)$ i. $(-5x^2y)(-3xy^2)$
    j. $(3g^3)(4g^4)$ k. $(9x^5)(9x^5)$ l. $4t^3 + 3t^4$

12. a. $(a^2b)^3$ b. $(xy^3)^2$ c. $(-2x)^4$ d. $(3a^2n^3)^3$

e. $(-3z^4)^3$ f. $2(a^2b)^3$ g. $-3(m^2n^2)^2$ h. $(-3m^2n^2)^2$

i. $(-cd^4)^3$ j. $(-x^2y)^4$ k. $\left(\frac{c^2}{d^3}\right)^2$ l. $\left(\frac{x}{y^4}\right)^2$

m. $\left(\frac{2x}{y^3}\right)^2$ n. $\left(\frac{-5x^2}{y^3}\right)^3$ o. $\left(\frac{-2u^3}{w}\right)^4$ p. $\left(\frac{(2x)^3}{x^3}\right)^4$

Review Problems

13. Evaluate each expression:

a. 2^4 b. 2^9 c. 2^{-2} d. 2^{-6}

e. 2^{-10} f. 10^3 g. 10^6 h. 10^{-4}

i. 10^{-7} j. 2^0 k. 10^0 l. $10^1 + 2^1$

m. $2^0 - 10^0$ n. $(2^3)^2$ o. $(10^4)^3$ p. $2^3 \cdot 2^4$

q. $10^2 \cdot 10^5$ r. $2^3 \cdot 10^2$ s. $2^3 + 10^2$ t. $2^{-1} + 2^{-2}$

u. $10^{-3} + 10^2$ v. $(10^{-2})^3$ w. $(2^{-3})^2$ x. $(10^{19})^0$

y. $(5^3)^0$ z. $(17^0)^{14}$

14. Simplify each expression:

a. $x^0 + x^1$ b. $x^2 \cdot x^0$ c. x^{-3} d. w^{-1}

e. $(ab)^1$ f. $(a + b)^0$ g. x^2x^4 h. $x^2 + x^4$

i. $x^4 - x$ j. x^3y^7 k. $a^5 + a^5$ l. a^5a^5

m. $\frac{x^3}{x}$ n. $\frac{a^6}{a^{10}}$ o. $(ab)^3$ p. $(xyz)^2$

q. $\left(\frac{x}{y}\right)^5$ r. $(x^2)^3$ s. $(u^3)^2$ t. $(w^3)^3$

u. $(A^{44})^0$ v. $(Q^0)^9$ w. $(2^5)^2$ x. $(2^2)^5$

y. $\frac{2^{10}}{2^9}$ z. $\frac{x^{10}}{x^9}$

15. Simplify each expression:

a. $(-3x^2)(4xy)$ b. $(-x^3w)(-2xw^3)$ c. $(2ab^4)^3$

d. $2(ab^4)^3$ e. $\left(\frac{x}{k^3}\right)^2$ f. $\frac{x^2}{k^3}$

g. $(2^{-1})(10^{-1})$ h. $\left(\frac{x^3}{2^{-5}}\right)^0$ i. $10^0 - 10^{-1}$

j. $2^0 + 3^0 + 4^0$ k. $2^{-1} + 2^{-2} + 2^{-3}$ l. $\frac{x^{100}}{x^{20}}$

Solutions

1. a. 17 b. 999 c. 3.8 d. -7 e. $\frac{2}{5}$ f. 1

g. 1 h. 1 i. $\frac{1}{3}$ j. $\frac{1}{16}$ k. $\frac{1}{125}$ l. $\frac{1}{1296}$

2. 1

3. a. x b. 1 c. $\frac{1}{n^3}$ d. $\frac{1}{z^5}$ e. 1 f. a

g. 1 h. xy i. $\frac{1}{t}$ j. 2 k. 1 l. x

m. y n. 1 o. $x + 1$ p. $1 - b$ q. 1 r. 10

s. $\frac{1}{w^2}$ t. $\frac{1}{z}$ u. $\frac{1}{a^5}$ v. $\frac{1}{u^8}$ w. $\frac{1}{n^4}$ x. $\frac{1}{w^6}$

y. $\frac{1}{a^7}$ z. $\frac{1}{k^8}$

4. $0^{-2} = \frac{1}{0^2} = \frac{1}{0}$, which is undefined.

5. a. $n^3n^3 = (nnn)(nnn) = nnnnnn = n^6$

b. $xx^4 = x(xxxx) = xxxxx = x^5$

c. z^8 d. a^{12} e. As is f. $2w^9$ g. As is h. 0

6. a. $\frac{x^8}{x^2} = \frac{xxxxxxxx}{xx} = \frac{\cancel{xx}xxxxxx}{\cancel{xx}} = x^6$ b. 1

c. $\frac{z^7}{z^8} = \frac{zzzzzzz}{zzzzzzzz} = \frac{\cancel{zzzzzzz}}{\cancel{zzzzzzz}z} = \frac{1}{z}$ d. As is

7. a. $(uv)^3 = (uv)(uv)(uv) = uuuvvv = u^3v^3$

b. $a^2b^2c^2$ c. x^4y^4 d. $m^2n^2p^2q^2$

e. j^2k^2 f. axy g. 1

8. The statement is false; pick some numbers for x and y, plug them into each side of the statement, and you'll see why. [See Chapter 2.]

9. a. $\left(\frac{a}{b}\right)^2 = \left(\frac{a}{b}\right)\left(\frac{a}{b}\right) = \frac{aa}{bb} = \frac{a^2}{b^2}$ b. $\frac{w^3}{z^3}$ c. $\frac{x^4}{y^4}$

d. $\left(\frac{1}{n}\right)^5 = \left(\frac{1}{n}\right)\left(\frac{1}{n}\right)\left(\frac{1}{n}\right)\left(\frac{1}{n}\right)\left(\frac{1}{n}\right) = \frac{1\cdot1\cdot1\cdot1\cdot1}{nnnnn} = \frac{1}{n^5}$

e. $\frac{u}{v}$ f. 1 g. $\frac{x^5}{32}$ h. $a^2 - 10a + 25$

10. a. $(2^3)^2 = (2\cdot2\cdot2)^2 = (2\cdot2\cdot2)\,(2\cdot2\cdot2) = 64$

b. 729 c. 1 d. 0 e. x^8

f. $(a^0)^4 = 1^4 = 1$ g. 1 h. 1

11. a. $6x^2$ b. $-6a^3$ c. $10y^6$ d. x^6

e. As is f. $17n^5$ g. 0 h. $-6xy^2$

i. $15x^3y^3$ j. $12g^7$ k. $81x^{10}$ l. As is

12. a. a^6b^3 b. x^2y^6 c. $16x^4$ d. $27a^6n^9$

e. $-27z^{12}$ f. $2a^6b^3$ g. $-3m^4n^4$ h. $9m^4n^4$

i. $-c^3d^{12}$ j. x^8y^4 k. $\frac{c^4}{d^6}$ l. $\frac{x^2}{y^8}$

m. $\frac{4x^2}{y^6}$ n. $\frac{-125x^6}{y^9}$ o. $\frac{16u^{12}}{w^4}$ p. 4096

13. a. 16 b. 512 c. $\frac{1}{4}$ d. $\frac{1}{64}$

e. $\frac{1}{1024}$ f. 1000 g. 1,000,000 h. $\frac{1}{10{,}000}$

i. $\frac{1}{10{,}000{,}000}$ j. 1 k. 1 l. 12

m. 0 n. 64 o. 1,000,000,000,000

	p.	128	q.	10,000,000	r.	800	s.	108
	t.	$\frac{3}{4}$	u.	100.001	v.	$\frac{1}{1{,}000{,}000}$	w.	$\frac{1}{64}$
	x.	1	y.	1	z.	1		

14.

a.	$1 + x$	b.	x^2	c.	$\frac{1}{x^3}$	d.	$\frac{1}{w}$
e.	ab	f.	1	g.	x^6	h.	$x^2 + x^4$
i.	$x^4 - x$	j.	x^3y^7	k.	$2a^5$	l.	a^{10}
m.	x^2	n.	$\frac{1}{a^4}$	o.	a^3b^3	p.	$x^2y^2z^2$
q.	$\frac{x^5}{y^5}$	r.	x^6	s.	u^6	t.	w^9
u.	1	v.	1	w.	1024	x.	1024
y.	2	z.	x				

15.

a.	$-12x^3y$	b.	$2x^4w^4$	c.	$8a^3b^{12}$	d.	$2a^3b^{12}$
e.	$\frac{x^2}{k^6}$	f.	As is	g.	$\frac{1}{20}$	h.	1
i.	$\frac{9}{10}$	j.	3	k.	$\frac{7}{8}$	l.	x^{80}

❑ **To ∞ and Beyond!**

Do some research to determine the meaning of $9^{1/2}$.

"Nothing can stop the man with the right mental attitude from achieving his goal; nothing on earth can help the man with the wrong mental attitude."

Thomas Jefferson