CH 62 – FACTORING QUADRATICS, AN INTRODUCTION

❑ *INTRODUCTION*

We're now ready to undertake the task of factoring additional quadratic expressions. This skill will not only enable us to solve many quadratic equations without resorting to the Quadratic Formula, but will allow us to reduce more kinds of algebraic fractions.

To set the stage for factoring binomials (two terms) and trinomials (three terms), we'll do one more example of double distributing to analyze its inner workings:

$$(2x + 3)(x + 5)$$

i) Multiply the FIRST terms in each set of parentheses: $2x$ and x. The product is $\underline{2x^2}$.

ii) Multiply the OUTER terms: $2x$ and 5. The product is $\underline{10x}$.

iii) Multiply the INNER terms: 3 and x. The product is $\underline{3x}$.

iv) Multiply the LAST terms in each set of parentheses: 3 and 5. The product is $\underline{15}$.

v) Writing out these four products, followed by combining like terms, gives us

$$\underset{\text{First}}{2x^2} + \underbrace{\underset{\text{Outer}}{10x} + \underset{\text{Inner}}{3x}} + \underset{\text{Last}}{15}$$

$$= \mathbf{2x^2 + 13x + 15}$$

To reiterate, we have

$$(2x + 3)(x + 5)$$

$=$	$2x^2$	$+$	$10x$	$+$	$3x$	$+$	15
	↑		↑		↑		↑
	product of First terms		product of Outer terms		product of Inner terms		product of Last terms

$$= \mathbf{2x^2 + 13x + 15}$$

The key idea to absorb here is that the $2x^2$ in the answer is the product of the first terms, while the 15 is the product of the last terms. And even more importantly, the middle term in the answer, $13x$, is the sum of the outer and inner products.

Homework

1. Find the following products -- do all the work in your head:

   a. $(x + 2)(x + 3)$ b. $(n - 3)(n - 5)$

   c. $(a + 7)(a - 5)$ d. $(b - 3)(b + 10)$

   e. $(y + 6)(y - 6)$ f. $(z + 3)(z + 3)$

   g. $(u + 1)(u + 7)$ h. $(v + 7)(v - 7)$

   i. $(x + 8)(x + 8)$ j. $(k - 10)(k - 10)$

2. Find the following products -- do all the work in your head:

   a. $(2x - 1)(x + 4)$ b. $(3n - 3)(2n - 5)$

   c. $(a + 9)(3a - 1)$ d. $(6y - 1)(2y + 7)$

   e. $(2m - 7)(2m + 7)$ f. $(4w + 5)(4w + 5)$

   g. $(5x + 1)(5x - 1)$ h. $(2n + 3)(7n - 10)$

   i. $(7u - 3)(7u - 3)$ j. $(12a + 13)(12a - 13)$

❑ UNSCRAMBLING THE EGG

Now suppose that we've been handed the quadratic expression

$$2x^2 + 13x + 15$$

to factor. We've done enough double distributing to realize that the factorization probably looks like this:

(<u>slot 1</u> + <u>slot 2</u>) (<u>slot 3</u> + <u>slot 4</u>)

where the contents of the four slots have to be determined.

We know that the product of slot 1 and slot 3 must be $2x^2$. It's logical to assume that slot 1 is $2x$ and slot 3 is x. Now we have

($2x$ + <u>slot 2</u>) (x + <u>slot 4</u>)

We also know that the product of slot 2 and slot 4 must be 15. Now we're at a crossroads: The product of 3 and 5 is 15, but the product of 1 and 15 is also 15. It gets worse -- even when we select a pair of factors of 15 to try, the <u>order</u> in which we place those factors in the slots might be crucial. As we try various combinations of numbers, keep in mind that we're trying to factor $2x^2 + 13x + 15$.

Let's try 1 and 15 (in that order):

$$(2x + \mathbf{1})(x + \mathbf{15}) = 2x^2 + 30x + x + 15 = 2x^2 + 31x + 15$$ ☹

Flip around the 1 and the 15:

$$(2x + \mathbf{15})(x + \mathbf{1}) = 2x^2 + 2x + 15x + 15 = 2x^2 + 17x + 15$$ ☹

That rules out using 1 and 15. Let's use the 5 and the 3 (in that order):

$$(2x + \mathbf{5})(x + \mathbf{3}) = 2x^2 + 6x + 5x + 15 = 2x^2 + 11x + 15$$ ☹

Flip the 5 and 3:

$$(2x + \mathbf{3})(x + \mathbf{5}) = 2x^2 + 10x + 3x + 15 = 2x^2 + 13x + 15$$ ☺

And so the factorization of $2x^2 + 13x + 15$ is $\mathbf{(2x + 3)(x + 5)}$.

Recall that we can summarize the process we just completed in two ways: In changing

$$2x^2 + 13x + 15 \quad \text{to} \quad (2x + 3)(x + 5)$$

we have

1) converted an expression whose final operation is addition into an expression whose final operation is multiplication, and we have
2) converted an expression with three terms into an expression with one term.

Either way, we've accomplished the goal of **factoring**.

Homework

3. True/False:

a. $x^2 + 5x + 6$ factors into $(x + 3)(x + 2)$.

b. $y^2 - 16$ factors into $(y + 4)(y - 4)$.

c. $x^2 + 9x + 15$ factors into $(x + 3)(x + 5)$.

d. $n^2 - 6n + 9$ factors into $(n - 3)(n - 3)$.

e. $2a^2 - 11a - 6$ factors into $(2a + 1)(a - 6)$.

f. $u^2 + 25$ factors into $(u + 5)(u + 5)$.

g. $a^2 + 10a + 25$ factors into $(a + 5)^2$.

h. $y^2 - 2y + 4$ factors into $(y - 2)^2$.

4. Matching:

a. ___ $x^2 + 4x + 3$

b. ___ $x^2 - 9$

c. ___ $x^2 + 10x + 25$

d. ___ $x^2 + 16$

e. ___ $x^2 + 4x + 4$

f. ___ $2x^2 - 11x - 6$

g. ___ $x^2 - 25$

h. ___ $2x^2 + x - 6$

1. $(2x + 1)(x - 6)$
2. $(x + 5)(x - 5)$
3. $(x + 3)(x + 1)$
4. $(x + 3)(x - 3)$
5. $(2x - 3)(x + 2)$
6. $(x + 5)(x + 5)$
7. $(x + 2)(x + 2)$
8. None of them

5. Finish the factorization of each expression:

a. $x^2 - 10x + 16 = (x - 8)(\qquad)$

b. $n^2 + 5n - 14 = (n + 7)(\qquad)$

c. $a^2 - 17a + 72 = (a - 9)(\qquad)$

d. $q^2 - 49 = (q + 7)(\qquad)$

e. $c^2 + 6c + 9 = (c + 3)(\qquad)$

6. Finish the factorization of each expression:

a. $12n^2 + 8n - 15 = (6n - \quad)(2n + \quad)$

b. $16x^2 - 9 = (4x + \quad)(4x - \quad)$

c. $21z^2 - 4z - 1 = (7z + \quad)(3z - \quad)$

d. $9a^2 + 24a + 16 = (3a + \quad)(3a + \quad)$

e. $6x^2 - 23x + 7 = (2x - \quad)(3x - \quad)$

f. $25t^2 - 49 = (5t + \quad)(5t - \quad)$

g. $9w^2 - 225 = (3w + \quad)(3w - \quad)$

h. $16c^2 - 24c + 9 = (4c - \quad)(4c - \quad)$

i. $14x^2 - 58x + 8 = (7x - \quad)(2x - \quad)$

7. Which of the following is the factorization of $x^2 - 2x - 8$?

   a. $(x + 8)(x - 1)$ b. $(x - 8)(x + 1)$

   c. $(x + 2)(x - 4)$ d. $(x - 2)(x + 4)$

8. Which of the following is the factorization of $3n^2 + 7n - 10$?

   a. $(3n - 1)(n + 10)$ b. $(3n + 5)(n - 2)$ c. $(3n - 5)(n + 2)$

   d. $(3n - 10)(n + 1)$ e. $(3n + 2)(n - 5)$ f. $(3n + 10)(n - 1)$

9. Your friend shows you her attempt at factoring:

   $$6x^2 + 17x - 14 = (2x + 1)(3x - 14)$$

   You seriously doubt the accuracy of her answer and suggest that perhaps she's made a mistake. She gets a little miffed and tells you, "Look! $2x$ times $3x$ is $6x^2$ and 1 times -14 is -14. See? It all works out." How might you gently show her the error of her ways?

Review Problems

10. Find the product in your head: $(3n - 7)(4n + 7)$

11. True/False: $16x^2 - 2x - 3$ factors into $(8x + 3)(2x - 1)$.

12. Finish the factorization of $14a^2 - 11a - 3$: $(a - 1)(\qquad)$

13. Finish the factorization of $10n^2 - 31n + 15$: $(5n \qquad)(2n \qquad)$

Solutions

1. a. $x^2 + 5x + 6$ b. $n^2 - 8n + 15$ c. $a^2 + 2a - 35$
d. $b^2 + 7b - 30$ e. $y^2 - 36$ f. $z^2 + 6z + 9$
g. $u^2 + 8u + 7$ h. $v^2 - 49$ i. $x^2 + 16x + 64$
j. $k^2 - 20k + 100$

2. a. $2x^2 + 7x - 4$ b. $6n^2 - 21n + 15$ c. $3a^2 + 26a - 9$
d. $12y^2 + 40y - 7$ e. $4m^2 - 49$ f. $16w^2 + 40w + 25$
g. $25x^2 - 1$ h. $14n^2 + n - 30$ i. $49u^2 - 42u + 9$
j. $144a^2 - 169$

3. a. T b. T c. F d. T e. T f. F g. T h. F

4. a. 3 b. 4 c. 6 d. 8 e. 7 f. 1 g. 2 h. 5

5. a. $x - 2$ b. $n - 2$ c. $a - 8$ d. $q - 7$ e. $c + 3$

6. a. 5, 3 b. 3, 3 c. 1, 1 d. 4, 4 e. 7, 1
f. 7, 7 g. 15, 15h. 3, 3 i. 1, 8

7. c.

8. f.

9. Everything she said was right -- she just didn't complete the process. You would point out (gently, remember) that the product of the outer terms, the $2x$ and the -14, is $-28x$, while the product of the inner terms, the 1 and the $3x$, is $3x$. Combining these two like terms produces $-25x$ for the middle term, not the required $17x$ that's in the original problem.

10. $12n^2 - 7n - 49$

11. True

12. $14a + 3$

13. $-3 \quad -5$

"The best thing for being sad," replied Merlin, beginning to puff and blow, "is to learn something. That's the only thing that never fails. You may grow old and trembling in your anatomies, you may lie awake at night listening to the disorder of your veins, you may miss your only love, you may see the world about you devastated by evil lunatics, or know your honour trampled in the sewers of baser minds. There is only one thing for it then — to learn. Learn why the world wags and what wags it. That is the only thing which the mind can never exhaust, never alienate, never be tortured by, never fear or distrust, and never dream of regretting. Learning is the only thing for you. Look what a lot of things there are to learn."

– T.H. White, *The Once and Future King* –