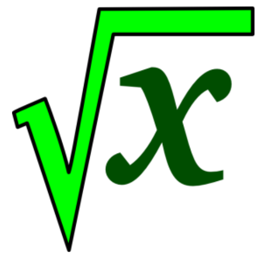
CH 70 – SOLVING QUADRATICS BY TAKING SQUARE ROOTS

❑ INTRODUCTION



Let's look at the quadratic equation

$$x^2 = 100$$

One way to solve it is by **factoring**:

$$x^2 = 100 \;\Rightarrow\; x^2 - 100 = 0 \;\Rightarrow\; (x + 10)(x - 10) = 0$$

$$\Rightarrow\quad x + 10 = 0 \text{ or } x - 10 = 0 \;\Rightarrow\; x = -10 \text{ or } x = 10$$

The solutions of the quadratic equation $x^2 = 100$ are simply $x = \pm\mathbf{10}$.

A second way is to use the **Quadratic Formula**:

$x^2 = 100 \Rightarrow x^2 - 100 = 0$, which means that $a = 1$, $b = 0$, and $c = -100$.
Placing these three values into the Quadratic Formula gives:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-0 \pm \sqrt{(0)^2 - 4(1)(-100)}}{2(1)} = \frac{\pm\sqrt{400}}{2} = \frac{\pm 20}{2} = \pm\,\mathbf{10}$$

But there's a third way (and easier, in this case) to solve a quadratic equation like $x^2 = 100$. Just take the square root of each side of the equation, <u>remembering that the number 100 has **two** square roots</u>, namely 10 and –10. Therefore, $x = \pm\mathbf{10}$, and neither factoring nor the Quadratic Formula is required for this simple quadratic equation.

For another example, let's solve the quadratic equation $n^2 = 30$. Remembering that 30 has two square roots, we calculate n to be $\pm\sqrt{\mathbf{30}}$.

For a third example, where we will need to simplify the radical, consider the quadratic equation $y^2 = 72$. When we take the square root of each side of the equation -- *and when we remember that* 72 *has* <u>*two*</u> *square roots* -- we see that

$$y^2 = 72 \quad \Rightarrow \quad y = \pm\sqrt{72} = \pm\sqrt{36\cdot 2} = \pm\sqrt{36}\cdot\sqrt{2} = \pm 6\sqrt{2}$$

In short, the solutions of $y^2 = 72$ are $\boldsymbol{\pm 6\sqrt{2}}$.

❑ *The Square Root Theorem*

Now for the general statement:

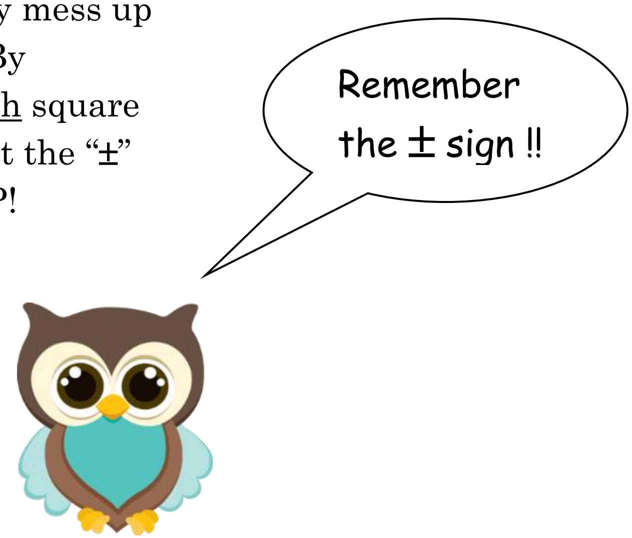
The solutions of the equation $x^2 = A$ are $x = \pm\sqrt{A}$

The Square Root Theorem

<u>Notes:</u>

1) The value of A in the Square Root Theorem is assumed to be zero or positive; this is, $A \ge 0$. Otherwise, the square root will not be a number, and therefore will be of no use to us in Elementary Algebra.

2) How do students usually mess up this kind of equation? By forgetting to include <u>both</u> square roots (that is, they forget the "±" sign). DON'T MESS UP!



<u>EXAMPLE 1:</u> **Solve the quadratic equation: $(x + 7)^2 = 81$**

<u>Solution:</u> According to the Square Root Theorem, we can remove the squaring by taking the square <u>root</u> of both sides of the equation, remembering that the number 81 has <u>two</u> square roots:

	$(x+7)^2 = 81$	(the original equation)
$\Rightarrow$	$x+7 = \pm\sqrt{81}$	(the Square Root Theorem)
$\Rightarrow$	$x+7 = \pm 9$	($\sqrt{81} = 9$)
$\Rightarrow$	$x = -7 \pm 9$	(subtract 7 from each side)

Using the plus sign yields $x = -7 + 9 = 2$.

Using the minus sign yields $x = -7 - 9 = -16$.

$$\boxed{x = 2 \text{ or } -16}$$

<u>EXAMPLE 2:</u> **Solve for y: $(y - 3)^2 = 32$**

<u>Solution:</u> First we need to remove, or undo, the squaring in this quadratic equation. This is where we apply the Square Root Theorem:

$y - 3 = \pm\sqrt{32}$ (32 has <u>two</u> square roots)

To isolate the y, add 3 to both sides:

$y = 3 \pm \sqrt{32}$

Since $\sqrt{32} = \sqrt{16 \times 2} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$, we simplify our solution:

$$\boxed{y = 3 \pm 4\sqrt{2}}$$

Be sure it's clear to you that we have written <u>two</u> solutions to our quadratic equation:

$3 + 4\sqrt{2}$ and $3 - 4\sqrt{2}$

<u>EXAMPLE 3:</u> **Solve for n: $(n-3)^2 = -49$**

<u>Solution:</u> Applying the Square Root Theorem to remove the squaring gives us the equation

$$n-3 = \pm\sqrt{-49}$$

We needn't go any further; after all, the square root of a negative number doesn't exist in this class. So we're done right here, and we conclude that the equation has

$$\boxed{\text{No Solution}}$$

Homework

1. Solve each equation by applying the Square Root Theorem:

a. $x^2 = 144$	b. $y^2 = 51$	c. $z^2 = 72$
d. $a^2 = 0$	e. $b^2 = -9$	f. $(x+1)^2 = 25$
g. $(n-3)^2 = 100$	h. $(u+10)^2 = 1$	i. $(a-5)^2 = 32$
j. $(b+7)^2 = 50$	k. $(w+13)^2 = -4$	l. $(m-3)^2 = 75$

❑ *SPLITTING RADICALS IN DIVISION*

Do you remember the rule about splitting the square root of a product:

$\sqrt{ab} = \sqrt{a}\sqrt{b}$? The same kind of rule works for division. For example,

we know that $\sqrt{\frac{9}{25}} = \frac{3}{5}$, since $\left(\frac{3}{5}\right)^2 = \frac{9}{25}$. Now let's work it out by

"splitting" the radical:

$$\sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}\text{, the same answer!}$$

In short, $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$ -- assuming, of course, that $b \neq 0$ and that $\dfrac{a}{b}$ is not negative.

Homework

2. Calculate each square root by "splitting" the radical:

   a. $\sqrt{\dfrac{1}{25}}$ b. $\sqrt{\dfrac{9}{49}}$ c. $\sqrt{\dfrac{16}{81}}$ d. $\sqrt{\dfrac{100}{121}}$ e. $\sqrt{\dfrac{144}{36}}$

We conclude this chapter with an example which requires us to "split" the square root of a fraction.

<u>EXAMPLE 4:</u> **Solve for z:** $\left(z - \dfrac{4}{5}\right)^2 = \dfrac{8}{25}$

<u>Solution:</u> Using the Square Root Theorem, we take the square root of each side of the equation, remembering that $\dfrac{8}{25}$ has <u>two</u> square roots:

$$z - \frac{4}{5} = \pm\sqrt{\frac{8}{25}} \qquad \text{(the Square Root Theorem)}$$

$$\Rightarrow \quad z = \frac{4}{5} \pm \sqrt{\frac{8}{25}} \qquad \text{(isolate the } z\text{)}$$

$$\Rightarrow \quad z = \frac{4}{5} \pm \frac{\sqrt{8}}{\sqrt{25}} \qquad \text{(split the radical)}$$

$$\Rightarrow \quad z = \frac{4}{5} \pm \frac{2\sqrt{2}}{5} \qquad \text{(simplify both square roots)}$$

$$\Rightarrow \quad \boxed{z = \frac{4 \pm 2\sqrt{2}}{5}} \qquad \text{(combine into a single fraction)}$$

Again, note that we have found <u>two</u> solutions. They may be ugly, but they're both solutions.

Homework

3. Solve each equation by applying the Square Root Theorem:

a. $\left(x-\frac{1}{2}\right)^2 = \frac{3}{4}$ b. $\left(t+\frac{2}{3}\right)^2 = \frac{1}{9}$ c. $\left(z-\frac{4}{5}\right)^2 = \frac{19}{25}$

d. $\left(x+\frac{3}{5}\right)^2 = \frac{12}{25}$ e. $\left(b-\frac{9}{10}\right)^2 = \frac{81}{100}$ f. $\left(g-\frac{3}{7}\right)^2 = \frac{24}{49}$

Review Problems

4. Solve each equation by applying The Square Root Theorem:

a. $x^2 = 121$ b. $y^2 = 50$ c. $z^2 = 0$

d. $n^2 = -25$ e. $t^2 = 288$ f. $a^2 - 14 = 0$

g. $(x+1)^2 = 75$ h. $(c-3)^2 = 10$ i. $(x+10)^2 = -1$

j. $\left(w+\frac{1}{2}\right)^2 = \frac{3}{4}$ k. $\left(u-\frac{4}{3}\right)^2 = \frac{26}{9}$ l. $\left(a+\frac{8}{5}\right)^2 = \frac{32}{25}$

Solutions

1. a. $x = \pm 12$ b. $y = \pm\sqrt{51}$ c. $z = \pm 6\sqrt{2}$

d. $a = 0$ e. No solution f. $x = 4, -6$

g. $n = 13, -7$ h. $u = -9, -11$ i. $a = 5 \pm 4\sqrt{2}$

j. $b = -7 \pm 5\sqrt{2}$ k. No solution l. $m = 3 \pm 5\sqrt{3}$

2. a. $\sqrt{\frac{1}{25}} = \frac{\sqrt{1}}{\sqrt{25}} = \frac{1}{5}$ b. $\frac{3}{7}$ c. $\frac{4}{9}$ d. $\frac{10}{11}$ e. 2

3. a. $\left(x - \frac{1}{2}\right)^2 = \frac{3}{4} \Rightarrow x - \frac{1}{2} = \pm\sqrt{\frac{3}{4}} \Rightarrow x = \frac{1}{2} \pm \frac{\sqrt{3}}{\sqrt{4}} = \frac{1}{2} \pm \frac{\sqrt{3}}{2} = \frac{1 \pm \sqrt{3}}{2}$

b. $t = -\frac{1}{3}, -1$ c. $z = \frac{4 \pm \sqrt{19}}{5}$ d. $x = \frac{-3 \pm 2\sqrt{3}}{5}$

e. $b = \frac{9}{5}, 0$ f. $g = \frac{3 \pm 2\sqrt{6}}{7}$

4. a. $x = \pm 11$ b. $y = \pm 5\sqrt{2}$ c. $z = 0$

d. No solution e. $t = \pm 12\sqrt{2}$ f. $a = \pm\sqrt{14}$

g. $x = -1 \pm 5\sqrt{3}$ h. $c = 3 \pm \sqrt{10}$ i. No solution

j. $w = \frac{-1 \pm \sqrt{3}}{2}$ k. $u = \frac{4 \pm \sqrt{26}}{3}$ l. $a = \frac{-8 \pm 4\sqrt{2}}{5}$

"Learning
is a
treasure

that will follow
its owner
everywhere."

Chinese Proverb