CH 73 – THE QUADRATIC FORMULA, PART II

❑ *INTRODUCTION*

Way back in Chapter 55 we used the Quadratic Formula to solve quadratic equations like $6x^2 + 31x + 40 = 0$, whose solutions are $-5/2$ and $-8/3$. In fact, all of the answers back there were either positive or negative whole numbers, or fractions, or zero. You may have realized that the reason we obtained these kinds of answers was that the square root part of the Quadratic Formula $(\sqrt{b^2 - 4ac})$ always worked out to the square root of a perfect square, like $\sqrt{49}$.

But we know that there are square roots like $\sqrt{5}$. What would we do if we came up with a radical like that, assuming that no calculator is allowed? In this section, we'll see that there's really nothing special to worry about -- just work the formula. Recall the Quadratic Formula:

$$\text{If } ax^2 + bx + c = 0, \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EXAMPLE 1: **Solve for x: $2x^2 - 3x - 1 = 0$**

Solution: For this quadratic equation, $a = 2$, $b = -3$, and $c = -1$. Inserting these values into the Quadratic Formula gives

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{3 \pm \sqrt{9+8}}{4} = \boxed{\frac{3 \pm \sqrt{17}}{4}}$$

And that's it, because $\sqrt{17}$ cannot be simplified, nor can the fraction be reduced. If you'd like, the two solutions of the equation can be written

$$x = \frac{3+\sqrt{17}}{4},\ \frac{3-\sqrt{17}}{4}$$

Homework

1. Solve each quadratic equation:

a. $x^2 - 5x + 1 = 0$

b. $2n^2 + n - 7 = 0$

c. $3a^2 - 3a - 1 = 0$

d. $5t^2 + 7t + 1 = 0$

e. $5x^2 + 15x - 2 = 0$

f. $t^2 + t - 8 = 0$

g. $5h^2 - 13h = -5$

h. $7u^2 + u = 3$

i. $2k^2 + 7k = 8$

j. $2m^2 - 3 = -9m$

2. Solve for x: $-7x^2 + 5x + 1 = 0$

Hint: Although not necessary, it's traditional to always make a, the leading coefficient, positive. This way, your solutions will match those in the book. Since $a = -7$ in this quadratic equation, we can multiply (or divide) each side of the equation by -1, yielding $7x^2 - 5x - 1 = 0$. Now you do the rest.

❑ *MORE QUADRATIC EQUATIONS*

EXAMPLE 2: **Solve for x: $2x^2 - 8x + 3 = 0$**

Solution: The values of a, b, and c for use in the Quadratic Formula are

$a = 2 \qquad b = -8 \qquad c = 3$

So now we state the Quadratic Formula, insert the three values, and then do a lot of arithmetic and simplifying until the final answer is exact (no decimals) and as clean as possible.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{(The Quadratic Formula)}$$

$$= \frac{-(\mathbf{-8}) \pm \sqrt{(\mathbf{-8})^2 - 4(\mathbf{2})(\mathbf{3})}}{2(\mathbf{2})} \quad (a = 2 \quad b = -8 \quad c = 3)$$

$$= \frac{8 \pm \sqrt{64 - 24}}{4} \quad \text{(square and multiply)}$$

$$= \frac{8 \pm \sqrt{40}}{4} \quad \text{(finish the arithmetic)}$$

$$= \frac{8 \pm \sqrt{4 \cdot 10}}{4} \quad \text{(factor the radicand)}$$

$$= \frac{8 \pm 2\sqrt{10}}{4} \quad \text{(simplify the radical)}$$

$$= \frac{\cancel{2}(4 \pm \sqrt{10})}{\cancel{2} \cdot 2} \quad \text{(factor the numerator and reduce)}$$

We write our two solutions as

$$\boxed{x = \frac{4 \pm \sqrt{10}}{2}}$$

<u>EXAMPLE 3:</u> **Solve for y: $3y^2 = 13$**

<u>Solution:</u> We begin by subtracting 13 from each side of the equation to convert the equation into standard quadratic form:

$$3y^2 - 13 = 0$$

At this point we see that $a = 3$, $b = 0$, and $c = -13$. Applying the Quadratic Formula:

$$y = \frac{-\mathbf{0} \pm \sqrt{\mathbf{0}^2 - 4(\mathbf{3})(\mathbf{-13})}}{2(\mathbf{3})} = \frac{\pm\sqrt{0+156}}{6} = \frac{\pm\sqrt{156}}{6}$$

$$= \frac{\pm\sqrt{4 \cdot 39}}{6} = \frac{\pm\sqrt{4}\sqrt{39}}{6} = \frac{\pm 2\sqrt{39}}{6} = \frac{\pm 2\sqrt{39}}{2 \cdot 3} = \frac{\pm \cancel{2}\sqrt{39}}{\cancel{2} \cdot 3}$$

And we have our final answer (two of them!) in simplest form:

$$\boxed{y = \pm\frac{\sqrt{39}}{3}}$$

<u>EXAMPLE 4:</u> **Solve for u: $5u^2 = 9u$**

<u>Solution:</u> First convert to standard form: $5u^2 - 9u = 0$, from which we deduce that $a = 5$, $b = -9$, and $c = 0$. Plugging these three values into the Quadratic Formula gives:

$$u = \frac{-(\mathbf{-9}) \pm \sqrt{(\mathbf{-9})^2 - 4(\mathbf{5})(\mathbf{0})}}{2(\mathbf{5})} = \frac{9 \pm \sqrt{81-0}}{10} = \frac{9 \pm 9}{10}$$

No radicals remain, so that issue is settled; using the plus and minus signs separately yields the two solutions:

$$\frac{9+9}{10} = \frac{18}{10} = \frac{9}{5} \quad \text{and} \quad \frac{9-9}{10} = \frac{0}{10} = 0$$

Final answer: $\boxed{u = \frac{9}{5},\ 0}$

EXAMPLE 5: **Solve for n: $2n^2 = 3n - 4$**

Solution: Converting to standard form, the equation becomes

$$2n^2 - 3n + 4 = 0,$$

where $a = 2$, $b = -3$, and $c = 4$. The Quadratic Formula produces

$$n = \frac{-(\mathbf{-3}) \pm \sqrt{(\mathbf{-3})^2 - 4(\mathbf{2})(\mathbf{4})}}{2(\mathbf{2})} = \frac{3 \pm \sqrt{9 - 32}}{4} = \frac{3 \pm \sqrt{-23}}{4}$$

Remembering that $\sqrt{-23}$ is not a number in Elementary Algebra, we stop right here and declare that the given equation has

No solution

Recall that a quadratic equation where a is negative, such as $-3x^2 + 2x - 1 = 0$, is best solved by making a positive. This can be accomplished by multiplying each side of the equation by -1, producing the equation $3x^2 - 2x + 1 = 0$. Now when you apply the Quadratic Formula, your answer will look like the one in the chapter solutions.

Homework

3. Solve each quadratic equation:

a. $3x^2 - 6x - 1 = 0$

b. $2k^2 = 8k - 6$

c. $2n^2 - 10 = 0$

d. $y^2 = 49$

e. $14d^2 - 4d = 4$

f. $x^2 + 3x + 10 = 0$

g. $5a^2 + 7a - 2 = 0$

h. $u^2 = 40$

i. $3z^2 = 3z$

j. $5x^2 - x + 1 = 0$

k. $x(x - 3) = 7$

l. $y(y + 1) = y(y - 7) + 13$

4. Solve each quadratic equation:

a. $x^2 + 6x + 1 = 0$
b. $2y^2 + 7y = 2$
c. $-n^2 + 28 = 0$
d. $5d^2 = 7d + 1$
e. $x^2 = -4 - 5x$
f. $x^2 + 2x + 1 = 0$
g. $-3a^2 = 5 - 12a$
h. $2x^2 = 7$
i. $3z^2 = 12z$
j. $n^2 + 9 = 0$
k. $-4n^2 + 12n = 9$
l. $t^2 - 9 = 0$

Review Problems

5. Solve each quadratic equation:

a. $x^2 - x - 56 = 0$
b. $u^2 - 8u = 0$
c. $r^2 = 9$
d. $9k^2 + 49 = -42k$
e. $n^2 + 2n - 80 = 0$
f. $-3t^2 = 11t + 6$
g. $2m^2 - 7m = -6$
h. $8d^2 = 2$
i. $-3a^2 = 7a - 6$
j. $-8h^2 - 8h = 0$
k. $n^2 - 3n + 4 = 0$
l. $2p^2 - 3p - 5 = 0$
m. $-3m^2 - 8m + 3 = 0$
n. $-8z^2 + 2z = 0$
o. $-3w^2 + 3 = 0$
p. $5g^2 = 5g + 2$
q. $-3y^2 + 6y = -3$
r. $12t^2 - 8t - 16 = 0$
s. $3x^2 = -15x + 3$
t. $0 = 21g^2 + 9g - 15$

u. $-5z^2 + 4z = -7$

v. $8q^2 + 4q = 3$

w. $y^2 - 4y = -2$

x. $2x^2 - 2x - 3 = 0$

y. $0 = 14d^2 - 4d - 4$

z. $27x^2 - 6x - 15 = 0$

Solutions

1. a. $\frac{5 \pm \sqrt{21}}{2}$ b. $\frac{-1 \pm \sqrt{57}}{4}$ c. $\frac{3 \pm \sqrt{21}}{6}$ d. $\frac{-7 \pm \sqrt{29}}{10}$

e. $\frac{-15 \pm \sqrt{265}}{10}$ f. $\frac{-1 \pm \sqrt{33}}{2}$ g. $\frac{13 \pm \sqrt{69}}{10}$ h. $\frac{-1 \pm \sqrt{85}}{14}$

i. $\frac{-7 \pm \sqrt{113}}{4}$ j. $\frac{-9 \pm \sqrt{105}}{4}$

2. $\frac{5 \pm \sqrt{53}}{14}$

3. a. The quadratic equation $3x^2 - 6x - 1 = 0$ is in standard form. We can read the values $a = 3$, $b = -6$, and $c = -1$. Placing these values into the Quadratic Formula gives

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(-1)}}{2(3)} = \frac{6 \pm \sqrt{36+12}}{6} = \frac{6 \pm \sqrt{48}}{6} = \frac{6 \pm \sqrt{16 \cdot 3}}{6}$$

$$= \frac{6 \pm 4\sqrt{3}}{6} = \frac{2(3 \pm 2\sqrt{3})}{2 \cdot 3} = \frac{\cancel{2}(3 \pm 2\sqrt{3})}{\cancel{2} \cdot 3} = \frac{3 \pm 2\sqrt{3}}{3}$$

b. 1, 3 c. $\pm\sqrt{5}$ d. ± 7

e. $\frac{1 \pm \sqrt{15}}{7}$ f. No solution g. $\frac{-7 \pm \sqrt{89}}{10}$ h. $\pm 2\sqrt{10}$

i. 0, 1 j. No solution k. $\frac{3 \pm \sqrt{37}}{2}$ l. $\frac{13}{8}$

4. a. $-3 \pm 2\sqrt{2}$ b. $\frac{-7 \pm \sqrt{65}}{4}$ c. $\pm 2\sqrt{7}$ d. $\frac{7 \pm \sqrt{69}}{10}$

e. $-1, -4$ f. -1 g. $\frac{6 \pm \sqrt{21}}{3}$ h. $\pm\frac{\sqrt{14}}{2}$

i. 0, 4 j. No solution k. $\frac{3}{2}$ l. ± 3

5. a. $-7, 8$ b. 8, 0 c. ± 3 d. $-\frac{7}{3}$

e. $-10, 8$ f. $-3, -\frac{2}{3}$ g. $2, \frac{3}{2}$ h. $\frac{1}{2}, -\frac{1}{2}$

i. $-3, \frac{2}{3}$ j. $-1, 0$ k. No solution l. $\frac{5}{2}, -1$

m. $-3, \frac{1}{3}$ n. $0, \frac{1}{4}$ o. ± 1 p. $\frac{5 \pm \sqrt{65}}{10}$

q. $1 \pm \sqrt{2}$ r. $\frac{1 \pm \sqrt{13}}{3}$ s. $\frac{-5 \pm \sqrt{29}}{2}$ t. $\frac{-3 \pm \sqrt{149}}{14}$

u. $\frac{2 \pm \sqrt{39}}{5}$ v. $\frac{-1 \pm \sqrt{7}}{4}$ w. $2 \pm \sqrt{2}$ x. $\frac{1 \pm \sqrt{7}}{2}$

y. $\frac{1 \pm \sqrt{15}}{7}$ z. $\frac{1 \pm \sqrt{46}}{9}$

“Do, or do not.

There is no 'try'.”

- Yoda (*The Empire Strikes Back*)