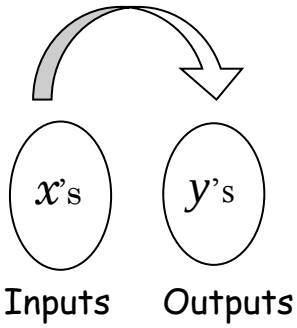
CH 88 – FUNCTIONS: FORMULAS AND GRAPHS

❑ *INTRODUCTION*

The previous chapter gave us the basic notion of a **function**: an entity where each input produces exactly one output. In this chapter we look at functions once again, this time using the more useful versions of functions: formulas and graphs.

❑ *FORMULAS*

When a function is written as a formula with x's and y's, we usually assume that the x's are the inputs and the y's are the outputs.



EXAMPLE 1:

A. $y = 2x + 7$ is a function.

Given an input x, there is exactly one output y. For example, if $x = 4$, then $y = 15$ and nothing else. [You may recall that this would be called a *linear* function, and its graph is a straight line.]

B. $y = x^4$ is a function.

Given an input x, there is exactly one output y. For instance, $x = 3$ produces $y = 81$. If $x = 0$, then $y = 0$. And if $x = -3$, then the unique output is 81. Remember that even though 81 is the output for two different inputs, the fact

still remains that $x = 3$ has a unique output of 81, and $x = -3$ also has a unique output of 81. So, for any value of x, the formula produces exactly one value of y. This is why it's a function.

C. $y = -3x^2 + 7x - 19$ is a function. [In fact, it's a *quadratic* function.]

D. $y = |x|$ is a function, called an *absolute value* function.

If $x = 7$, then $y = 7$ -- exactly one output for the input of 7.
If $x = -7$, then $y = 7$ -- exactly one output for the input of -7.

E. $x = |y|$ is not a function.

Why isn't it a function? We need to conjure up some input (an x) which has more than one output (y). Let's choose $x = 4$. This gives us the equation $4 = |y|$, and there are two solutions to this equation; namely, y could be 4, or y could be -4. We have more than one output for a single input. It follows that this formula does not represent a function.

F. $3x - 7y = 19$ is a function.

Here we could solve for y,

$$y = \frac{3}{7}x - \frac{19}{7},$$

and see that given an x, there's only one y for it, so this is a (linear) function.

G. $x^2 + y^2 = 4$ is not a function.

Let $x = 0$. Then $y^2 = 4 \Rightarrow y = \pm 2$. That is, an input of 0 results in two outputs, 2 and -2. Thus, this formula (which you'll discover next semester is a circle) is not a function.

H. $y = \pm\sqrt{x}$ is <u>not</u> a function.

If we choose 81 as the input, we get two outputs: ± 9, violating the fundamental notion of a function.

I. $y = 5$ <u>is</u> a function.

Given any input (the x), there's only one output, namely 5. Therefore, this horizontal line is a function.

J. $x = 2$ is <u>not</u> a function.

In fact, this example probably holds the world's record for being a non-function. After all, given an input (the only choice being 2), there are an <u>infinite</u> number of outputs (y can be <u>anything</u>). We thus see that a vertical line is not a function.

x	y
2	5
2	0
2	–3

Homework

1. Consider the formula $y = x^3$. Recall that x is an input and y is an output. When a value of x has been assigned, there's only one y value. What does this mean?

2. Consider $y^2 = x$. When $x = 25$, y has two values: 5 and –5. What does this mean?

3. Consider $x = |y + 10|$.

   a. When $x = 20$, y has two values. Verify this fact.

   b. What does this mean?

4. Determine whether or not the given formula is a function:

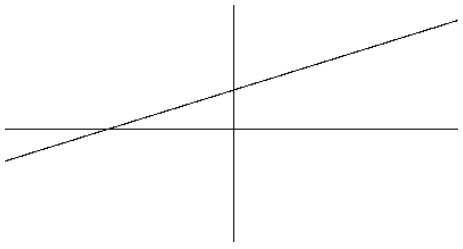
a. $y = -7x + 9$	b. $x = 2y + 5$	c. $y = x^5$
d. $y^2 = x$	e. $x^2 + y^2 = 14$	f. $y = \lvert 7x + 2 \rvert$
g. $x^2 - y^2 = 9$	h. $x = \pi$	i. $y = \sqrt{2}$
j. $y = \pm\sqrt{x-1}$	k. $y = \frac{x+1}{x-3}$	l. $y = \sqrt{2x^2 + x + 1}$
m. $x = \lvert y + 5 \rvert$	n. $x^2 + y^2 = 1$	o. $y = \sqrt{1-x}$
p. $y = x^3 - x^2 + x$	q. $x - 5 = 0$	r. $y + 2\pi = 0$
s. $3x - 7y = 8$	t. $y = \lvert 10 - x \rvert$	u. $x = \lvert 1 - y \rvert$

❑ *GRAPHS*

Now it's time to determine whether a given *graph* is a function or not. We will assume that the standard x- and y-axes are used, and as before, we agree that x is the input and y is the output. Recalling that a function must produce exactly one output for each legal input, we look at the following three graphs.

EXAMPLE 2:

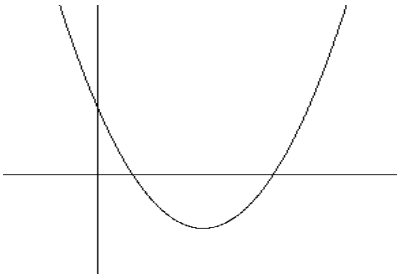
A.



Choose any x-value on the x-axis -- this is the input. Now go straight up or down until you get to the graph to find the y-value -- this is the output. How many outputs did you get? You should have gotten exactly one output. Whatever

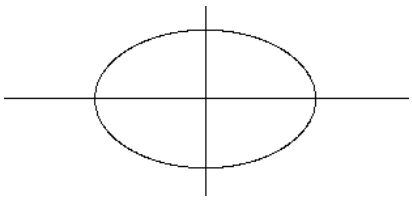
legal x you use, there's exactly one y for it. The graph <u>is</u> a function.

B.



Pick any input on the x-axis. Go up or down until you hit the graph -- the y-value is the output. Every legal value of x produces exactly one output. This graph <u>is</u> also a function.

C.



Choose a legal input (say, $x = 0$). Now go find the graph. This time we run into the graph <u>twice</u>, once going up and once going down. We have an x which has two different y's. That is, we have a legal input with more than one output. This graph is definitely <u>not</u> a function.

The following chart indicates other examples of graphs and their status as functions:

FUNCTIONS	NON-FUNCTIONS
single point	vertical line
horizontal line	circle
top-half of a circle	ellipse (oval)
Non-vertical line	right-half of a circle
parabola opening up	parabola opening right

Homework

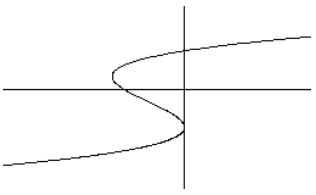
5. T/F: No line is a function.

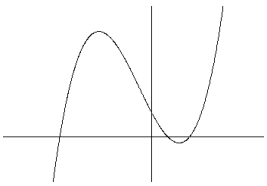
6. Which of the following are functions?

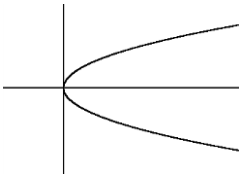
   a. horizontal line b. bottom half of a circle c. left half of a circle

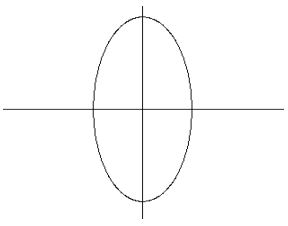
7. Explain why a graph consisting of a single point is a function.

8. Which of the following are functions?

a. 

b. 

c. 

d. 

Review Problems

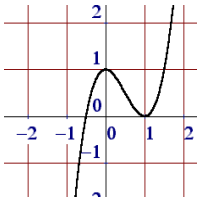
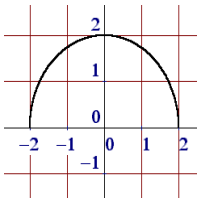
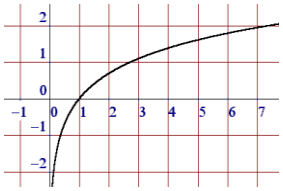
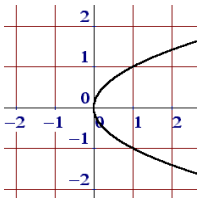
9. Consider the formula $x^2 + y^2 = 1$. Explain why it's not a function.

10. Consider the formula $y = \pm\sqrt{x}$. If $x = 9$, what is y? Is this a function?

11. Is it a function?

a. $x = \lvert y + 1 \rvert$	b. $y = x^3$	c. $y = \pm\sqrt{x-3}$
d. $x = 5$	e. $y = -\pi$	f. $x^2 = y^2$
g. $7x - 9y = 10$	h. $y = \lvert x + 5 \rvert$	i. $y = 7x^2 - \pi x^{10}$
j. $x^2 + y^2 = 49$	k. $x = 0$	l. $y = 0$

12. Which are functions?

13. True/False:

a. $x = y^4$ is a function.

b. $y = \lvert x - 2 \rvert$ is a function.

c. $y = \pm\sqrt{x-\pi}$ is a function.

d. $x = 9$ is a function.

e. $y = -\pi$ is a function.

f. $x^2 + y^2 = 2$ is a function.

g. Every line is a function.

h. Every semicircle is a function.

i. $x = \lvert y - 1 \rvert$ is a function.

Solutions

1. It means that the formula represents a function, since each input has a unique output.

2. It means that the formula does <u>not</u> represent a function, since there's an input with more than one output.

3. a. If $x = 20$, then $20 = |y+10|$. This implies that

$y + 10 = 20$ or $y + 10 = -20$, giving two solutions: $y = 10, -30$.

b. One input (20) produced two outputs (10 and –30). Therefore, the formula does <u>not</u> represent a function.

4. a. Yes, given an input (x), there's only one output (y).
b. Yes, solve for y and it's just like part a.
c. Yes, one input produces exactly one output.
d. No, an x-value of 4 produces two y-values, namely 2 and –2.
e. No, an x-value of 0 produces two y-values.
f. Yes
g. No, if $x = 10$, then y has two values.
h. No, for $x = \pi$, there are an infinite number of y's.
i. Yes, given any input, the output must be $\sqrt{2}$, so there's a unique output for each input.
j. No, if $x = 9$, then y has two values, 2 and –2.
k. Yes, put in any legal value of x, and only one y-value will result.
l. Yes, it may be complicated but only one y-value will appear for a given x-value.
m. No, if $x = 2$, then y can be either –3 or –7.
n. No, let $x = 0$ and see what you get.
o. Yes, as long as an appropriate x is chosen, the positive square root produces exactly one answer.
p. Yes q. No r. Yes s. Yes t. Yes u. No

5. F (actually, most lines are functions)

6. a. and b.

7. When you choose the only legal x, you're already at the y-value, and there's only one.

8. b. only

9. If $x = 0$, $y = \pm 1$; an input has produced two outputs, so it's not a function. Or, the graph is a circle, which is certainly not a function.

10. $y = \pm 3$; it is not a function, since a single input of 9 produced two outputs.

11. a. No b. Yes c. No d. No e. Yes f. No
g. Yes h. Yes i. Yes j. No k. No l. Yes

12. All but the 4th

13. a. F b. T c. F d. F e. T f. F
g. F h. F i. F

"Ninety-nine percent of the failures come from people who have the habit of making excuses."

– George Washington