
–THE ALGEBRA GLOSSARY–

– A –

Absolute Value

The distance from a number to 0 on the number line, and denoted with vertical bars. Examples:

$$|7| = 7 \qquad | -6 | = 6 \qquad |0| = 0$$

Notice that, since distance is never negative, the absolute value of a number must be greater than or equal to 0:

For any number n , it is always the case that $|n| \geq 0$.

Acute Angle

An angle greater than 0° but less than 90° . The two non-right angles in a right triangle must be acute angles.

Addition Method for a System of Equations

The *addition method* is a way to solve a *system of equations* by adding the equations together in such a way that one of the variables is eliminated, and then solving for the remaining variable. For example let's use the addition (elimination) method to solve the system of equations:

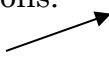
$$\begin{aligned} 5x - 2y &= 20 \\ 3x + 7y &= -29 \end{aligned}$$

2

In the addition method we may eliminate either variable. But there's a certain orderliness that comes in handy in future math courses if we always eliminate the first variable in each equation, so in this case we will eliminate the x . Before we add the equations, however, we might need to multiply one or both equations by some numbers, and then add the resulting equations to kill off one of the variables, and then solve for the variable that still lives. How do we find these numbers? Rather than some mystifying explanation, let's just do an example-- you'll catch on.

$$\begin{array}{rcl} 5x - 2y = 20 & \xrightarrow{\text{times } 3} & 15x - 6y = 60 \\ 3x + 7y = -29 & \xrightarrow{\text{times } -5} & -15x - 35y = 145 \end{array}$$

Add the equations: $0x - 41y = 205$

The x 's are gone! 

Divide by -41 :
$$\frac{-41y}{-41} = \frac{205}{-41}$$
$$y = -5$$

Now that we have the value of y , we can substitute its value of -5 into either of the two original equations to find the value of x . Using the first equation:

$$\begin{aligned} 5x - 2(-5) &= 20 \\ \Rightarrow 5x + 10 &= 20 \\ \Rightarrow 5x &= 10 \\ \Rightarrow x &= 2 \end{aligned}$$

Therefore, the final solution to the system of equations is

$x = 2 \text{ \& } y = -5$

Addition Property of Equations

This is the term we use to justify the fact that the same quantity can be added to both sides of an equation without changing the solution(s) of that equation.

Additive Identity

The technical term given to the number zero. We know, for instance, that $7 + 0 = 7$. When we add zero to a number, the *sum* is identical to the number. See Zero.

Additive Inverse

Means the same thing as “opposite.” For example, since $8 + (-8) = 0$, we call -8 the additive inverse of 8, since adding them produces the *additive identity*, zero.

Area

The size of a 2-dimensional region enclosed by a geometric shape. The amount of carpet needed for a room would be based on the room’s area.

The area of a *rectangle* with length l and width w : $A = lw$

The area of a *square* with side s : $A = s^2$

The area of *circle* with radius r : $A = \pi r^2$

Asset Allocation

A type of word problem which attempts to distribute money to various investments according to given criteria.

Associative Property (Law)

For any numbers,

$$(a + b) + c = a + (b + c) \quad \text{and} \quad (ab)c = a(bc)$$

Example for addition:

$$\begin{array}{ll} (2 + 3) + 4 & 2 + (3 + 4) \\ = 5 + 4 & = 2 + 7 \\ = 9 & = 9 \quad \checkmark \end{array}$$

Example for multiplication:

$$\begin{array}{ll} (2 \times 3) \times 4 & 2 \times (3 \times 4) \\ = 6 \times 4 & = 2 \times 12 \\ = 24 & = 24 \quad \checkmark \end{array}$$

Axis (plural: axes)

In a 2-dimensional coordinate system, the x -axis is the horizontal axis, and the y -axis is the vertical axis.

– B –

Base

In the expression x^2 , x is the base and 2 is the exponent (or power).

Binary Number System

As opposed to the *decimal number system*, which is based on 10, the binary number system is based on 2, and is the basis for all computer operations and every device in our culture that we call “digital.”

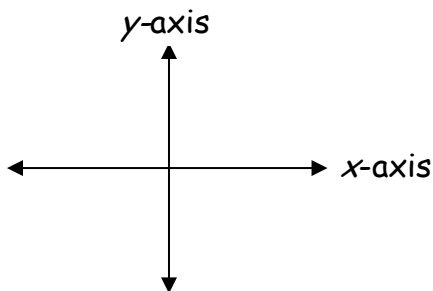
Binomial

A *polynomial* with two terms. $3x + 7$ and $5n^2 - 7n$ are typical examples.

– C –

Cartesian Coordinate System

In Elementary Algebra, it's the 2-dimensional x - y coordinate system where equations are graphed. Named in honor of the French mathematician/philosopher René Descartes (1596–1650).



Celsius

The temperature scale, devised by Anders Celsius, where the freezing point of water is 0°C and the boiling point of water is 100°C. The formulas for converting between Celsius and Fahrenheit are

$$C = \frac{F - 32}{1.8} \qquad F = 1.8C + 32$$

Circle

If $r = \text{radius}$, $d = \text{diameter}$, $C = \text{circumference}$, and $A = \text{area}$, then

$$d = 2r \qquad r = \frac{d}{2} \qquad C = 2\pi r \qquad A = \pi r^2$$

See *Pi*.

Circumference

The distance around a circle (the circle's perimeter).

$C = 2\pi r$, where r is the radius. An equivalent formula: $C = \pi d$

Closure

When two whole numbers are multiplied, the product is definitely another whole number. We say that the whole numbers are closed under multiplication. But when two whole numbers are divided, the quotient may not be a whole number; for example, 4 divided by 12 is $\frac{1}{3}$, which is not a whole number. We say that the whole numbers are not closed under division.

Coefficient

It's "how many" there are of something. If we have 8 oranges, the coefficient is 8. The coefficient of the expression $12x^2$ is 12, because that's how many x^2 s we have.

Combining Like Terms

"3 bananas" and "12 bananas" are *like terms*. We can add them together to get a sum of "15 bananas."

$7x$ and $3x$ are like terms and can be combined into a single term:

$$7x + 3x = 10x$$

$7a$ and $3y$ are not like terms, so they cannot be combined into a single term (apples and oranges), but $3n^2$ and $-10n^2$ are like terms:

$$3n^2 - 10n^2 = -7n^2$$

Common Factor

A factor (multiplier) that is contained in two or more terms. For example, $2x$ is a common factor of the trinomial $16x^3 - 8x^2 + 4x$ (although it is not the *greatest common factor*).

Commutative Property (Law)

Statement: For any numbers a and b ,

$$a + b = b + a \quad \text{and} \quad ab = ba$$

Examples: $3 + 2 = 2 + 3$ and $7 \times 9 = 9 \times 7$

Note: There is no commutative law for subtraction or division, since, for example,

$$9 - 7 \neq 7 - 9 \quad \text{and} \quad \frac{8}{4} \neq \frac{4}{8}.$$

Complete Factoring

Factoring until there's nothing left to factor. The expression $4x^2 - 4$ factors into $4(x^2 - 1)$, but it's not complete. Continuing the factoring gives us $4(x + 1)(x - 1)$.

Composite Number

A *natural number* greater than 1 that is not *prime*. Therefore, a composite number has more than two factors. For example, 15 is a composite number since it has four factors: 1, 3, 5, and 15. Composite numbers can always be written as a *product of primes*. For example, $20 = 2 \times 2 \times 5$.

Constant

Something that doesn't change in a problem. Obvious constants are 7 and π ; they never change. Even letters like a and b can be constants in a problem, if it's assumed that they represent numbers that don't change in the problem. See *Variable*.

Coordinate

Each of the numbers in an ordered pair (point in the plane) is a coordinate. For example, for the point (10, 3), 10 is the x -coordinate and 3 is the y -coordinate.

Counterexample

A single example which disproves a general statement. For example, consider the statement “All prime numbers are odd.” This is an overwhelmingly valid statement . . . with one exception: the number 2. It’s a prime number (the smallest), but it’s even. This exception to the statement “All prime numbers are odd” is called a counterexample, and thus makes the statement false (even though it’s “mostly” true).

Cross-Cancel

In working the problem $\frac{a}{b} \times \frac{b}{c}$, we can write $\frac{ab}{bc}$, and then reduce it to $\frac{a}{c}$. Or, we can “cross-cancel” the b ’s and be done with it. We can also cross-cancel with units of measurement. For example, to convert 6 miles into yards, and knowing only that 5,280 ft = 1 mi and that 3 ft = 1 yd, we can calculate in the following way:

$$\frac{6 \text{ mi}}{1} \times \frac{5,280 \text{ ft}}{1 \text{ mi}} \times \frac{1 \text{ yd}}{3 \text{ ft}} = \frac{6 \cancel{\text{mi}}}{1} \times \frac{5,280 \cancel{\text{ft}}}{1 \cancel{\text{mi}}} \times \frac{1 \text{ yd}}{3 \cancel{\text{ft}}} = \frac{31,680}{3} \text{ yd} = 10,560 \text{ yd}$$

Cube

In geometry, it’s a box where all six sides are squares. In algebra, the cube of n is written n^3 , which means $n \cdot n \cdot n$. For example, the cube of 5 is $5 \cdot 5 \cdot 5 = 125$. We call n^3 “ n -cubed” because if all the edges of a cube have a measure of n units, then the volume of the cube is given by the formula $V = n^3$.

– D –

Decimal Number System

The number system we generally use to do math (computers use other systems). It uses the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The set of all decimals is also called the set of *real numbers*, denoted \mathbb{R} .

Degree

A unit of angle measurement. Abbreviated “°”, it’s defined so that once around a circle is 360° . This means that a straight angle is 180° , and a right angle is 90° .

Temperature, both in *Fahrenheit* and *Celsius*, is measured in degrees.

Also, a *polynomial* has a degree: for example, the degree of $3x^4 - 7x + 6$ is 4.

Delta

The capital Greek letter “delta” is written Δ , and is used in math and science to represent a change in something. For example, a change in pressure might be written ΔP . The main use of the delta in algebra is the slope formula: the change in y over the change in x , $\frac{\Delta y}{\Delta x}$.

Denominator

The bottom of a fraction. Be sure it’s never 0. See *Zero*.

Diameter

The diameter of a circle is the distance from one point on a circle to another point on the circle, passing through the center of the circle. Note that the diameter of a circle is always twice its radius: $d = 2r$.

Difference

The result of subtracting two quantities. The difference of 10 and 2 is 8, and the difference of x and y is written $x - y$.

Difference of Two Squares

An expression of the form $A^2 - B^2$. This expression factors to $(A + B)(A - B)$. For example, $25x^2 - 16y^2$ is a difference of squares [think of it as $(5x)^2 - (4y)^2$].

Distributive Property (Law)

We know from arithmetic that multiplication is just repeated addition. For example,

$$3 \times 5 = 5 + 5 + 5$$

In the same manner,

$$3w = w + w + w$$

We also recall the combining of like terms,

$$4x + 2x = 6x$$

We now combine these two ideas to create the most important property of numbers in all of mathematics. Here's an example:

$$\begin{aligned} 4(x + 5) &= x + 5 + x + 5 + x + 5 + x + 5 && \text{(repeated addition)} \\ &= x + x + x + x + 5 + 5 + 5 + 5 && \text{(rearrange the terms)} \\ &= 4x + 20 && \text{(combine like terms)} \end{aligned}$$

Leaving out all the stretching and combining, we have shown that

$$4(x + 5) = 4x + 20$$

In other words, just multiply the 4 by the x , and then multiply the 4 by the 5.

In general, for any numbers a , b , and c , the Distributive Property says that

$$a(b + c) = ab + ac$$

The a is distributed to both the b and the c .

Examples:

$$7(x + 10) = 7x + 70$$

$$7n(2n - 3) = 14n^2 - 21n$$

$$9(3x - 2y + z) = 27x - 18y + 9z$$

$$-5x^2(3x - 1) = -15x^3 + 5x^2$$

The distributive property can also be used in reverse to *factor* an expression. Two examples:

$$5x + 15 = 5(x + 3)$$

$$14x^2 + 21x = 7x(2x + 3)$$

Dividend

It's the part of a division problem that the divisor is being "divided into."

$$\frac{\text{dividend}}{\text{divisor}} \qquad \text{dividend} \div \text{divisor} \qquad \text{divisor} \overline{) \text{dividend}}$$

Division by Zero

Absolutely forbidden; it's undefined.

Examples:

$$\frac{0}{9} = 0 \qquad \frac{6}{0} \text{ is undefined} \qquad \frac{0}{0} \text{ is undefined}$$

See *Zero*.

Divisor

It's the part of a division problem that's "dividing into" the dividend.

$$\frac{\text{dividend}}{\text{divisor}} \qquad \text{dividend} \div \text{divisor} \qquad \text{divisor} \overline{) \text{dividend}}$$

Does Not Exist

In Elementary Algebra, it's a phrase used to describe the result of trying to take the square root of a negative number. For example, $\sqrt{-25}$ does not exist as a number in Elementary Algebra, and here's the reason.

We're trying to find a number, which when squared, gives an answer of -25 . In other words, we're seeking a number n such that $n^2 = -25$. If n is a positive number, then n^2 is also positive, and if n is negative, then n^2 is still positive. And, of course, $0^2 = 0$. The square of a number (in Elementary Algebra) can never be negative, so $\sqrt{-25}$ does not exist.

Double

To multiply a number by 2. The result of doubling 10 is 20, and the result of doubling n is $2n$. The double of a whole number is always an *even number*.

Double Distributive

A phrase to help you remember how to multiply two *binomials* together:

$$(a + b)(c + d) = ac + ad + bc + bd$$

For example,

$$\begin{aligned}
 (3n + 7)(2n - 10) &= 6n^2 - 70n + 14n - 70 \\
 &= 6n^2 - 56n - 70
 \end{aligned}$$

– E –

Electron

A tiny charged particle revolving around the nucleus of an atom. Its charge is -1 . The “sum” of a *proton* (whose charge is $+1$) and an electron is 0 ; they cancel each other out. That’s why, in algebra, $1 + (-1) = 0$.

Elimination Method for a System of Equations

See *Addition Method*.

Equation

Two expressions set equal to each other, usually with the goal of finding the value of one of the variables that would make the two sides of the equation equal. For example, $7x - 1 = 10$ and $3n^2 - 8n = 23$ are both equations. See *Expression*.

Equation of a Line

The slope-intercept form of a line is

$$y = mx + b,$$

where m is the slope and $(0, b)$ is the y -intercept. For example, the line $y = 2x - 7$ has a slope of 2 and a y -intercept of $(0, -7)$. [Some teachers say that the y -intercept is $y = 7$.]

Equilateral Triangle

A triangle with three equal sides (and therefore three equal angles, 60° each). An equilateral triangle is a special case of an *isosceles triangle*, since an isosceles triangle has two or more equal sides.

Evaluating an Expression

We can evaluate an expression of constants like this:

$$4(9 - 12) = 4(-3) = -12$$

Or we can evaluate an expression with a variable if the value of the variable is known. For example, if $x = 10$, then we can evaluate the expression $x^2 + 9$ like this:

$$x^2 + 9 = 10^2 + 9 = 100 + 9 = 109$$

Even Number

An *integer* divisible by 2. Every even number is twice some integer; e.g., $-14 = 2(-7)$. Thus, every even number can be written in the form $2n$, where n is some integer. If we assume that x is an even number, then the next even number is $x + 2$.

Exponent

In the expression x^3 , x is called the base and 3 is called the exponent, or the power. y^2 can be read “ y to the second power, or “ y squared,” and means $y \times y$.

x^3 can be read “ x to the third power, or “ x cubed,” and means $x \cdot x \cdot x$. Thus, $5^2 = 25$ and $4^3 = 64$.

Zero Exponent

Exponent Laws

| <u>LAW</u> | <u>EXAMPLE</u> |
|-----------------------------|----------------------------|
| $x^a x^b = x^{a+b}$ | $a^5 a^3 = a^8$ |
| $\frac{x^a}{x^b} = x^{a-b}$ | $\frac{w^{10}}{w^2} = w^8$ |
| $(x^a)^b = x^{ab}$ | $(z^4)^5 = z^{20}$ |
| $(xy)^n = x^n y^n$ | $(xy)^4 = x^4 y^4$ |

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n} \qquad \left(\frac{g}{h}\right)^9 = \frac{g^9}{h^9}$$

Expression

An expression is simply a glob of math stuff, without an equal sign. For example, $\frac{10+3}{9-1}$ and $8x^2 - 7y$ are expressions. When two expressions (with at least one variable involved) are set equal to each other, we get an *equation*. For example, whereas $7x + 3$ and $9x - 10$ are expressions, $7x + 3 = 9x - 10$ is an equation.

— F —

Factor (noun)

A quantity which divides evenly (without remainder) into another quantity. Example: 5 is a factor of 35. So are 1, 7 and 35. See *Prime*. Another example: n is a factor of $3n^2$.

Factor (verb)

Factoring is the process of converting an expression with two or more *terms* into a single term. Equivalently, factoring is the process of converting an expression whose final operation is addition or subtraction into one whose final operation is multiplication.

$$\text{Examples: } 10x + 15y = 5(2x + 3y)$$

$$x^2 + 5x + 6 = (x + 3)(x + 2)$$

Fahrenheit

The temperature scale, devised by Daniel Fahrenheit, where the freezing point of water is 32°F and the boiling point of water is 212°F. The formulas for converting between *Celsius* and Fahrenheit are

$$C = \frac{F - 32}{1.8} \qquad F = 1.8C + 32$$

Equivalently,

$$C = \frac{5}{9}(F - 32) \qquad F = \frac{9}{5}C + 32$$

Formula

An equation with two or more variables in it. The area of a circle, $A = \pi r^2$ is a formula. The temperature conversion equations above are formulas. When a formula (such as $ax + b = c$) doesn't refer to anything in particular, it's sometimes called a literal equation.

Fractions

— G —

GCF

Stands for **G**reatest **C**ommon **F**actor. It's the largest possible factor that is contained in all of the terms of a given expression. Example: The GCF of $10x^2 + 15x$ is $5x$.

Geometry Formulas

P = perimeter C = circumference A = area V = volume

s = side of a square

l = length of a rectangle or box

w = width of a rectangle or box

r = radius of a circle, cylinder, sphere, or cone

h = height of a box, cylinder, or cone

Square: $P = 4s$ $A = s^2$

Rectangle: $P = 2l + 2w$ $A = lw$

Circle: $C = 2\pi r$ $A = \pi r^2$

Box: $A = 2lw + 2wh + 2lh$ $V = lwh$

Cylinder: $A = 2\pi r^2 + 2\pi rh$ $V = \pi r^2 h$

Sphere:

$$A = 4\pi r^2$$

$$V = \frac{4}{3}\pi r^3$$

Cone:

$$A = \pi r^2 + \pi r \sqrt{h^2 + r^2}$$

$$V = \frac{1}{3}\pi r^2 h$$

Graphing

Creating a picture from an equation, by plotting points that satisfy the equation. For example, the point (2, 7) would lie on the graph of the equation $y = 3x + 1$ because if $x = 2$ and $y = 7$ are placed into the equation $y = 3x + 1$, the result will be a true statement.

Greatest Common Factor

See *GCF*.

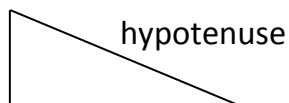
- H -

Horizontal Line

A flat line extending left to right (or west to east). Its equation is always of the form $y = \text{some number}$, and its slope must be 0. Example: The graph of $y = 7$ is a horizontal line.

Hypotenuse

The hypotenuse of a right triangle can be defined either as the longest side of the triangle, or as the side that is opposite the 90° angle. See *Leg*, *Pythagorean Theorem*.



– I –

Identity

This term has three meanings in Elementary Algebra.

1. An equation which is true for all possible values of the variables. For example, $(x + 3)(x - 3) = x^2 - 9$ is true for any value of x .
2. The number 0 is called the *additive identity*.
3. The number 1 is called the *multiplicative identity*.

Imaginary Number

A number which is not a *real number*; equivalently, a number that can never be written as a *decimal*. The only kind of imaginary number in Elementary Algebra is the square root of a negative number, e.g., $\sqrt{-144}$.

Inequality

A statement using one of the symbols $>$, $<$, \geq , or \leq . For example, $2x - 4 < 10$ is an inequality whose solution is $x < 7$.

Integer

The set of integers is the set we get from taking the *natural* numbers 1, 2, 3, 4, . . . and throwing in 0 and the opposites of the natural numbers. Thus, the set of integers is the set:

$$\{ \dots -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots \}$$

Integer Operations

$$7 + 2 = 9$$

$$9 - 3 = 6$$

$$7 - 10 = -3$$

$$-4 - 5 = -9$$

$$8 + (-3) = 5$$

$$(-12) + (-3) = -15$$

$$3 - (-4) = 3 + 4 = 7$$

$$-8 - (-3) = -8 + 3 = -5$$

$$(2)(3) = 6$$

$$(-3)(5) = -15$$

$$(20)(-2) = -20$$

$$(-4)(-7) = 28$$

$$\frac{6}{2} = 3$$

$$\frac{-10}{2} = -5$$

$$\frac{20}{-5} = -4$$

$$\frac{-8}{-2} = 4$$

Intercepts

In a 2-dimensional graph, the intercepts are the places on the x and y axes through which the graph passes.

Consider the line $2x - 3y = 12$. We can find one point on the line very easily by letting $x = 0$. This produces

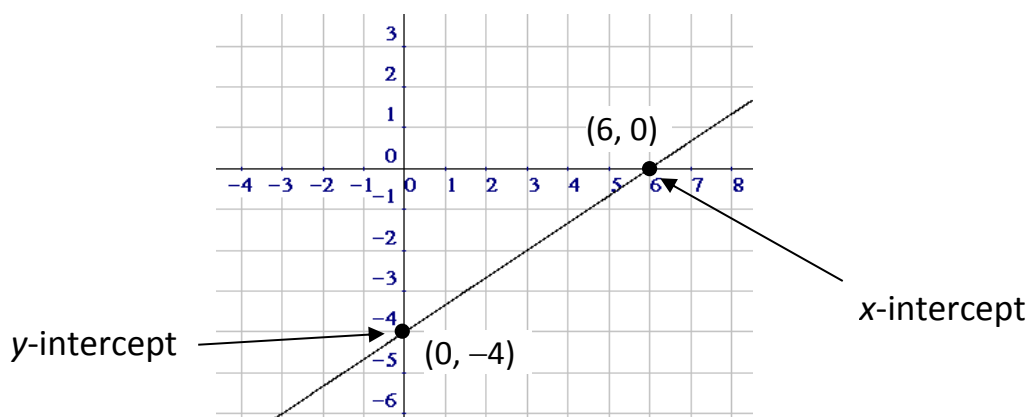
$$2(0) - 3y = 12 \Rightarrow 0 - 3y = 12 \Rightarrow -3y = 12 \Rightarrow y = -4$$

This shows us that the point $(0, -4)$ is on the line.

Now let's set y to 0. We obtain

$$2x - 3(0) = 12 \Rightarrow 2x - 0 = 12 \Rightarrow 2x = 12 \Rightarrow x = 6$$

We conclude that the point $(6, 0)$ is also on the line. Since two points suffice to construct a line (although plotting more than two points is an excellent idea!), we'll graph our line now using the points $(0, -4)$ and $(6, 0)$:



Notice that the point $(6, 0)$, although certainly on the line $2x - 3y = 12$, is special because it lies on the x -axis. We call the point $(6, 0)$ in this example the ***x-intercept*** of the line. Similarly, we call the point $(0, -4)$ the ***y-intercept*** of the line.

Looking back at the calculations, we see that $x = 6$ (which gave us the x -intercept) was found by setting y to 0, and $y = -4$ (which yielded the y -intercept) was found by setting x to 0. Here's a summary of this easy way to find the intercepts of a line (and other graphs, too):

To find x -intercepts, set $y = 0$

To find y -intercepts, set $x = 0$

Inverse

See *Additive Inverse* and *Multiplicative Inverse*. We also say that, for example, subtraction is the inverse operation of addition.

Isosceles Triangle

A triangle with at least two equal sides (and therefore at least two equal angles). Notice that if the isosceles triangle happens to have three equal sides, then we also call it an *equilateral triangle*.

— L —

Laws of Exponents

See *Exponent Laws*

LCD

See *Least Common Denominator*

LCM

See *Least Common Multiple*

Least Common Denominator

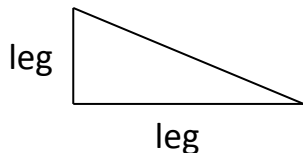
It's the *LCM* when dealing with fractions. For example, whereas the LCM of 10 and 15 is 30, the LCD of the denominators in the problem $\frac{1}{10} + \frac{7}{15}$ is 30.

Least Common Multiple

Given two quantities, it's the smallest quantity that each of the two given quantities divides into evenly. The LCD of 12 and 8 is 24, and the LCD of x^2y^5 and ax^3 is ax^3y^5 .

Leg

Either of the two short sides of a *right triangle*. In fact, the legs form the right angle. See *Hypotenuse*, *Pythagorean Theorem*.



Like Terms

Like terms contain exactly the same variable and its powers. For example, $7n$ and $3n$ are like terms. Also, $-4x^2$ and πx^2 are like terms. But $7Q$ and $8R$ are not like terms; $3y^2$ and $4y^3$ are not like terms. See *Combining Like Terms*.

Line

The graph obtained from one of the equations

| | |
|-----------------------|---------------------|
| $y = mx + b$ | (a slanted line) |
| $x = \text{a number}$ | (a vertical line) |
| $y = \text{a number}$ | (a horizontal line) |

Linear Equation

An equation in which all the exponents on the variables are 1. For example, $2(x - 3) = 7x + 9$ is a linear equation in one variable.

$y = \pi x + 8$ is a linear equation in two variables; the graph of this equation is a straight line -- hence the term "linear."

Linear Equation (Solving)

Here's the quintessential type of linear equation in Elementary Algebra:

$$\text{Solve for } x: \quad 2(3x - 7) - 5(1 - 3x) = -(-4x + 1) + (x + 7)$$

The steps to solve this equation are

- 1) Distribute
- 2) Combine like terms
- 3) Solve the simplified equation

$$2(3x - 7) - 5(1 - 3x) = -(-4x + 1) + (x + 7)$$

$$\begin{aligned} \Rightarrow 6x - 14 - 5 + 15x &= 4x - 1 + x + 7 && \text{(distribute)} \\ \Rightarrow 21x - 19 &= 5x + 6 && \text{(combine like terms)} \\ \Rightarrow 21x - \mathbf{5x} - 19 &= 5x - \mathbf{5x} + 6 && \text{(subtract } 5x \text{ from each side)} \\ \Rightarrow 16x - 19 &= 6 && \text{(simplify)} \\ \Rightarrow 16x - 19 + \mathbf{19} &= 6 + \mathbf{19} && \text{(add 19 to each side)} \\ \Rightarrow 16x &= 25 && \text{(simplify)} \\ \Rightarrow \frac{16x}{\mathbf{16}} &= \frac{25}{\mathbf{16}} && \text{(divide each side by 16)} \\ \Rightarrow \boxed{x = \frac{25}{16}} &&& \text{(simplify)} \end{aligned}$$

Literal Equation

An equation with two or more variables in it. The formula for the area of a circle, $A = \pi r^2$, is a literal equation. See *Formula*.

— M —

Mean

A statistic found by dividing the sum of a set of numbers by how many numbers are in the set. For example, the *mean* of the data set

5, 7, 13, 2, 3

is

$$\frac{5+7+13+2+3}{5} = \frac{30}{5} = 6$$

Measurements

| | |
|---|---|
| 1 foot (ft) = 12 inches (in) 1 yard (yd) = 3 ft 1 mile (mi) = 5,280 ft 1 sq ft (ft ²) = 144 sq in (in ²) 1 sq yd (yd ²) = 9 ft ² | [1 <i>meter</i> (m) <i>is about 39.37 inches</i>] 1 kilometer (km) = 1,000 meters (m) equivalently, 1 m = 0.001 km 1 centimeter (cm) = 0.01 m equivalently, 1 m = 100 cm 1 millimeter (mm) = 0.001 m equivalently, 1 m = 1,000 mm |
| 1 pound (lb) = 16 ounces (oz) 1 ton = 2,000 lb | [1 <i>gram</i> (g) <i>is about 1/28 of an ounce</i>] 1 kilogram (kg) = 1,000 grams (g) equivalently, 1 g = 0.001 kg 1 milligram (mg) = 0.001 g equivalently, 1 g = 1,000 mg |
| 1 teaspoon (tsp) = 16 drops 1 tablespoon (Tbsp) = 3 tsp 1 cup = 16 Tbsp 1 cup = 8 fluid ounces (fl oz) 1 pint (pt) = 2 cups 1 quart (qt) = 2 pt 1 gallon (gal) = 4 qt | [1 <i>liter</i> (L) <i>is about 1.06 quarts</i>] 1 kiloliter (kL) = 1,000 L equivalently, 1 L = 0.001 kL 1 milliliter (mL) = 0.001 L equivalently, 1 L = 1,000 mL |
| 1 kg = 2.2 lb 1 L = 1.06 qt 1 in = 2.54 cm | A cube that is 1 cm on each side is called a cubic centimeter (cc). Things are defined so that 1 cc is the same volume as 1 mL: 1 cc = 1 mL |

Monomial

A polynomial consisting of one term.

Some Examples:

$17x^5$ It's a single term since the operation between the 17 and the x^5 is multiplication.

23 It can be thought of as $23x^0$.

$x^5 + x^2$ This is not a monomial because the final operation is addition, not multiplication. (But it is a *binomial*.)

Multiplication Property of Equations

This is the term we use to justify the fact that each side of an equation can be multiplied by the same (nonzero) quantity without changing the solution(s) of that equation.

Multiplicative Identity

The technical term given to the number 1, because, for example, $9 \times 1 = 9$. When we multiply a number by 1, the *product* is identical to the number.

Multiplicative Inverse

The fancy term for *reciprocal*.

— N —

Natural Numbers

The set of positive whole numbers: $\{1, 2, 3, 4, \dots\}$

Negative Exponent

A negative exponent denotes reciprocal. For example,

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9} \qquad \text{In general, } x^{-n} = \frac{1}{x^n}$$

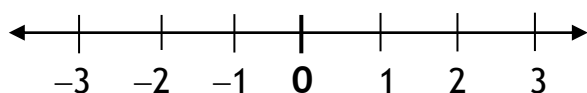
Negative Number

A *real number* that is less than zero. Every real number is either negative, zero, or positive. The *opposite* of a negative number is a *positive number*, while the *reciprocal* of a negative number is negative.

Neutron

A tiny, uncharged particle contained in the nucleus of an atom. Its charge is 0.

Number Line



Note that only the natural numbers, the opposites of the natural numbers, and zero are labeled on the number line. But there are infinitely more numbers on the number line. For example, $\frac{2}{3}$ and $-\pi$ are on the number line, too. Sometimes called the real number line.

Numerator

The top of a fraction, and is the dividend when the fraction is considered as a division problem. It can be 0, as long as the denominator is not 0. See *Zero*.



Odd Number

An *integer* that is not evenly divisible by 2. For example, 17 is an odd number. Every odd number is one more than twice some integer; e.g., $17 = 2(8) + 1$. Thus, every odd number can be written in the form $2n + 1$, where n is some integer. If we assume that x is an odd number, then the next odd number is $x + 2$. For instance, if $x = 23$, then the next odd number is $x + 2 = 23 + 2 = 25$.

Opposite

The opposite of x is $-x$ and the opposite of $-x$ is x . The opposite of 9 is -9 , the opposite of -7 is 7, and the opposite of 0 is 0. Every number has an opposite. On a number line, a number and its opposite are always the same distance from the origin 0. Note: The sum of a number and its opposite is always 0:

$$n + (-n) = 0$$

See *Additive Inverse*.

Optimization

A problem where we're trying to make something as big as possible or as small as possible (usually with some type of restriction, called a *constraint*). For example, if you had 200 ft of fencing, what shaped rectangle would you design so that you get the biggest possible area?

Order of Operations

This is a universal agreement regarding the order in which we carry out computations so that we all get the same answer. Almost all calculators and computer languages use this order. Here's the list from highest priority to lowest:

- 1) Parentheses () and Brackets [] and Braces { }
- 2) Exponents
- 3) Multiply and Divide (left to right)
- 4) Add and Subtract (left to right)

Examples:

| | |
|--|--------------------------------------|
| $7 - 9 + 2 = -2 + 2 = 0$ | (add and subtract, left to right) |
| $20 \div 2 \times 10 = 10 \times 10 = 100$ | (multiply and divide, left to right) |
| $10 + 3 \times 5 = 10 + 15 = 25$ | (multiply before add) |
| $3 \cdot 5^2 = 3 \cdot 25 = 75$ | (exponent before multiply) |
| $(10 - 7)^2 = 3^2 = 9$ | (parentheses before any operation) |
| $20 + 3[10 - (8 - 2)]$ | |
| $\quad = 20 + 3[10 - 6]$ | (innermost inclusion symbols first) |
| $\quad = 20 + 3[4]$ | |

$$= 20 + 12$$

$$= 32$$

Ordered Pair

A point in the plane is represented by the ordered pair (x, y) , where x is the horizontal distance and y is the vertical distance from the *origin*. For example, to plot the point $(4, -7)$, start at the origin $(0, 0)$, move 4 units to the right, and then 7 units down.

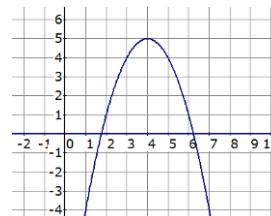
Origin

On a *number line*, it's 0. In an x - y coordinate system (the plane), it's the point $(0, 0)$, the point where the x - and y - axes intersect.

– P –

Parabola

The path taken when a football is thrown into the air. Also, satellite dishes are in the shape of a parabola. A typical equation of a parabola is $y = -x^2 + 8x - 11$.



Parallel Lines

Two lines (in the same plane) which never intersect. Therefore, they go in the same direction and they have equal slopes (assuming they're not vertical lines, since the slope of a vertical line is undefined).

Parentheses

Symbols used to group things together. For example, officially the expression

$$2 \times 3 + 4$$

would require multiplying 2 by 3 first. But if we really want the addition to be done first, we write

$$2 \times (3 + 4)$$

Percent Mixture Problems

$$\text{Quantity} \times \% \text{ Concentration} = \text{Actual Amount}$$

Perfect Square

A quantity which is the square of something else. For example, 49 is a perfect square because it's the square of 7. In addition, z^2 and $(a - b)^2$ are also perfect squares. Even x^6 is a perfect square because it's equal to $(x^3)^2$.

Perimeter

The total distance around a geometric object. For example, the length of baseboard needed for a room would be based on the room's perimeter. See *Square, Rectangle, Circle*.

Perpendicular

Two lines are perpendicular if they meet at a right angle (90°). Any horizontal line is perpendicular to any vertical line.

Pi

π (a letter of the Greek alphabet) is defined in math to be the ratio of the circumference of any circle to its diameter: $\pi = \frac{C}{d}$. It is an infinite, non-repeating decimal with approximations of 3.14 and $\frac{22}{7}$.

Plane

A flat, 2-dimensional geometric object. When the x - and y -axes are placed on a plane, it's then called a coordinate plane, which we called the *Cartesian coordinate system* in Elementary Algebra.

Point

A dimensionless position, represented by a real number x on the 1-dimensional *number line*, or by the *ordered pair* (x, y) in the 2-dimensional *plane* (or the ordered triple (x, y, z) in 3-dimensional space, or even the ordered quadruple (x, y, z, t) in 4-dimensional space-time).

Polynomial

A typical polynomial expression looks like

$$3x^5 - \pi x^3 + x^2 - 9x + \frac{4}{5}$$

A polynomial may consist of a single term:

$$17x^3$$

A linear polynomial is an expression of the form

$$mx + b$$

A quadratic polynomial is any expression of the form

$$ax^2 + bx + c \quad (\text{where } a, b, \text{ and } c \text{ are real numbers}).$$

The main theme of a polynomial is that all of the exponents on the x (or whatever variable) must be one of the whole numbers 0, 1, 2, 3, The following are not polynomials: $8x^{-2}$ and $3x^{1/2}$, because the exponents -2 and $1/2$ are not whole numbers. See *Monomial*, *Binomial*, and *Trinomial*.

Positive Number

A *real number* that is greater than 0. Every real number is either negative, zero, or positive. The positive whole numbers are called *natural numbers*. The *opposite* of a positive number is a negative number, while the *reciprocal* of a positive number is a positive number.

Power

A power is an exponent. x^2 is read “ x to the 2nd power” or “ x squared,” and means $x \cdot x$.

x^3 is read “ x to the 3rd power” or “ x cubed,” and means $x \cdot x \cdot x$.

x^4 is read “ x to the 4th power” and means $x \cdot x \cdot x \cdot x$.

The first few (non-negative) powers of 2 are 1, 2, 4, 8, 16, 32, 64, 128, 256, and 512.

The first few (non-negative) powers of 10 are 1, 10, 100, 1000, 10000, 100000, and 1000000.

Prime Factorization

Expressing a natural number (greater than 1) as a product of primes.

For example, $60 = 2^2 \cdot 3 \cdot 5$

Prime Number

A *natural number* greater than 1 whose only factors are 1 and itself. For example, 2 and 37 are prime numbers. 51 is not a prime number because it has factors of 3 and 17. Notice that since a prime is by definition greater than 1, it follows that 2 is the smallest prime; indeed, 2 is the only even prime.

Product

The result of multiplying two quantities. The product of 10 and 2 is 20, and the product of x and y is written xy [or $x \cdot y$, or $(x)(y)$]. The product of two numbers with the same sign is positive. The product of two numbers with different signs is negative.

Proportion

Two equal fractions: $\frac{a}{b} = \frac{c}{d}$. Multiplying each side of the equation by the LCD, which is bd , gives the equivalent equation $ad = bc$. A proportion can also be defined as the equality of two *ratios*.

Proton

A tiny charged particle contained in the nucleus of an atom. Its charge is +1. The “sum” of a proton and an *electron* (whose charge is -1) is 0; they cancel each other out. Since the charges of 8 protons and 8 electrons would cancel each out, we have in algebra that $8 + (-8) = 0$.

Pythagorean Theorem

If a and b are the *legs* of a *right triangle*, and c is the *hypotenuse*, then

$$a^2 + b^2 = c^2$$

Classic Example

If the legs of a right triangle are 5 and 12, how can we find the hypotenuse?

$$\begin{aligned} a^2 + b^2 &= c^2 \\ \Rightarrow 5^2 + 12^2 &= c^2 \\ \Rightarrow 25 + 144 &= c^2 \end{aligned}$$

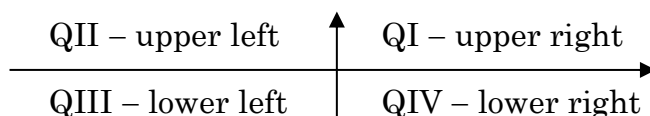
$$\Rightarrow 169 = c^2$$

$$\Rightarrow c = 13, \text{ and we conclude that } \underline{\text{the hypotenuse is 13.}}$$

— Q —

Quadrant

The four parts of the Cartesian Plane (our x - y coordinate system) which don't include the x and y axes. Quadrant I is the upper right region, and the rest of the quadrants proceed counterclockwise.



Quadratic Equation

An equation of the form $ax^2 + bx + c = 0$. The essential property of a quadratic equation is that the variable is squared. The equation $7x + x^2 = 9$ is also a quadratic equation, but is almost always converted to standard form: $x^2 + 7x - 9 = 0$. Three methods of solving a quadratic equation are *Factoring*, *Completing the Square*, and the *Quadratic Formula*.

Quadratic Formula

The two solutions (if you count *imaginary numbers*) of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Consider the radicand $b^2 - 4ac$.

If it's positive, the equation has two real solutions.

If it's zero, the equation has one real solution.

If it's negative, the equation has no real solutions.

Notice that a can never be 0 in this scenario, for two reasons:

1. If a were 0, the ax^2 term would drop away and it would no longer be a quadratic equation.

2. The Quadratic Formula would lead to division by 0, and we know we can't have that! See *zero*.

Quadruple

To multiply a number by 4. The result of quadrupling 10 is 40, and the result of quadrupling n is $4n$.

Quintuple

To multiply a number by 5. The result of quintupling 10 is 50, and the result of quintupling n is $5n$.

Quotient

The result of dividing two quantities. The quotient of 10 and 2 is 5, and the quotient of x and y is written $\frac{x}{y}$, or $x \div y$, or x / y . The quotient of two numbers with the same sign is positive. The quotient of two numbers with different signs is negative. The quotient $\frac{0}{4}$ equals 0. The quotients $\frac{8}{0}$ and $\frac{0}{0}$ are undefined. See *Zero*.

– R –

Radical

In the expression \sqrt{xy} , the $\sqrt{}$ is called the radical sign, and the xy is called the *radicand*.

Radicand

It's the stuff inside the radical sign. For example, the radicand of the quantity $\sqrt{2x^5}$ is $2x^5$.

Radius

On the one hand, a radius of a circle is any line segment connecting the center of a circle to any point on the circle. In this regard, we can see that a circle has an infinite number of radii. On the other hand, the more common

use of the term radius is the distance from the center of the circle to any point on the circle. The radius of a circle is always one-half its diameter: $r = \frac{d}{2}$.

Ratio

A comparison, using division, between two things. When we say that the Cougars scored twice as many points as the Bears, we are comparing their scores with a ratio.

If there are 20 boys and 15 girls, then the ratio of boys to girls, “20 to 15,” can be written 20:15, or $\frac{20}{15}$, each of which can be simplified to 4:3, or $\frac{4}{3}$.

However it’s written, the meaning is the same: For every 4 boys there are 3 girls. See *Proportion*.

Real Number

A real number is any number which either is a *decimal* or can be written as a decimal. The only type of number that might appear in Elementary Algebra that is not a real number is the square root of a negative number. For example, $\sqrt{-25}$ is not a real number, and is called an *imaginary number*. Here are some examples of real numbers:

$$7 = 7.0$$

$$0 = 0.0$$

$$-9 = -9.0$$

$$\frac{2}{3} = 0.66666\dots$$

$$-\frac{15}{4} = -3.75$$

$$\frac{23}{99} = 0.232323\dots$$

$$\pi = 3.14159265\dots$$

$$-\sqrt{2} = -1.41421356\dots$$

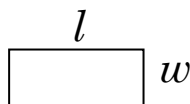
Reciprocal

The reciprocal of n is $\frac{1}{n}$. The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$. Zero has no reciprocal, since $\frac{1}{0}$ is undefined. Note that the product of a number and its reciprocal is always 1: $\frac{a}{b} \times \frac{b}{a} = 1$. The technical term for reciprocal is *multiplicative inverse*. To divide two fractions, we multiply the first fraction by the reciprocal of the second:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

Rectangle

A 4-sided figure with opposite sides equal and every angle equal to 90° .



$$\text{Perimeter} = 2l + 2w$$

$$\text{Area} = lw$$

A *square* is a special rectangle where all four sides have the same length. Notice, therefore, that every square is a rectangle, but certainly not every rectangle is a square.

Reducing Fractions

Process: Factor and divide out (or cancel) common factors: $\frac{ka}{kb} = \frac{a}{b}$

$$\text{Example: } \frac{10}{15} = \frac{2 \times 5}{3 \times 5} = \frac{2 \times \cancel{5}}{3 \times \cancel{5}} = \frac{2}{3}$$

$$\text{Example: } \frac{x^2 + 5x + 6}{x^2 - 4} = \frac{(x+2)(x+3)}{(x+2)(x-2)} = \frac{\cancel{(x+2)}(x+3)}{\cancel{(x+2)}(x-2)} = \frac{x+3}{x-2}$$

Right Angle

A 90° angle. Two lines at a right angle are *perpendicular* to each other. The wall and the ceiling are at a 90° angle.

Right Triangle

A triangle with a *right angle* (90°) in it. The two sides that form the right angle are the *legs* and the side opposite the right angle is the *hypotenuse*. See *Pythagorean Theorem*.

Rise

In the definition of *slope*, the rise is the change in y -values, and is the numerator of the fraction which defines slope: $m = \frac{\text{rise}}{\text{run}}$

Run

In the definition of *slope*, the run is the change in x -values, and is the denominator of the fraction which defines slope: $m = \frac{\text{rise}}{\text{run}}$

— S —

Signed Numbers

Positive and negative numbers (and zero), including fractions, decimals, and roots.

Simplify

This word has many meanings, the most important being the removal of parentheses and the combining of like terms. But there are other meanings, depending upon the situation. Here are some examples:

$$7(9 - 10) = 7(-1) = -7$$

$$2(x - 3) + 8x = 2x - 6 + 8x = 10x - 6$$

$$x^4 \cdot x^5 = x^9$$

$$\frac{15}{20} = \frac{3}{4}$$

$$\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2}$$

Simplifying Radicals

Bringing out of the radical as much as you can:

$$\sqrt{196} = 14$$

$$\sqrt{98} = \sqrt{49 \cdot 2} = \sqrt{49} \cdot \sqrt{2} = 7\sqrt{2}$$

Simultaneous Equations

Another name for *system of equations*.

Slope

The steepness of a line, denoted by the letter “ m ” and given by the equivalent formulas:

$$m = \frac{\text{rise}}{\text{run}} \qquad m = \frac{\text{change in } y}{\text{change in } x} \qquad m = \frac{\Delta y}{\Delta x}$$

See delta for an explanation of Δ .

The slope of the line through the points (a, b) and (c, d) is $m = \frac{b-d}{a-c}$.

The definition of slope is designed so that

1. the slope of a rising line is positive.
2. the slope of a falling line is negative.
3. the slope of any horizontal line is 0.
4. the slope of any vertical line is undefined.

Slope-Intercept Form of a Line

The equation of a line in the form $y = mx + b$, where m is the slope and $(0, b)$ is the y -intercept. This form works for all lines except vertical ones.

Solution

Usually refers to the answer to an equation or system of equations. For example, the solution of the equation $y + 3 = 10$ is $y = 7$ (though some would say that the solution is 7).

Square

In geometry, it's a rectangle with four equal sides. If s is the length of each side of a square, then the perimeter is $P = 4s$ and the area is $A = s^2$.

In algebra, the square of n is written n^2 , which means $n \cdot n$. For example, the square of 8 is 64.

Square Root

Assuming n is at least zero (i.e., greater than or equal to 0), then a square root of n is a number whose square is n . For example, 10 is a square root of 100, since $10^2 = 100$. But -10 is also a square root of 100, because $(-10)^2 = 100$. We conclude that 100 has two square roots. The two square roots of 100 can also be written as $\sqrt{100}$ and $-\sqrt{100}$. The number 2 has two

square roots, written $\sqrt{2}$ and $-\sqrt{2}$, although these square roots are non-repeating decimals with an infinite number of digits. $\sqrt{2}$ is a *real number*, while $\sqrt{-2}$ is an *imaginary number*.

The Basic Rule of Square Roots:

$$\sqrt{a}\sqrt{b} = \sqrt{ab}$$

Reading this rule from left to right, we can multiply square roots:

$$\sqrt{7} \times \sqrt{10} = \sqrt{70}$$

Reading from right to left, we can simplify square roots:

$$\sqrt{72} = \sqrt{36 \cdot 2} = \sqrt{36} \cdot \sqrt{2} = 6\sqrt{2}$$

Squaring a Binomial

| | |
|-------------------------|---------------------------------|
| $(a + b)^2$ | |
| $= (a + b)(a + b)$ | $(n^2 = (n)(n))$ |
| $= a^2 + ab + ba + b^2$ | (First, Inner, Outer, Last) |
| $= a^2 + ab + ab + b^2$ | (multiplication is commutative) |
| $= a^2 + 2ab + b^2$ | (combine like terms) |

Substitution Method

A method of solving a *system of equations*, where a variable is solved for in one equation, and then its value is substituted into another equation.

Sum

The result of adding two quantities. The sum of 10 and 2 is 12, and the sum of x and y is written $x + y$.

System of Equations

More than one equation with more than one variable. A typical example is

$$\begin{aligned} 2x + 3y &= 10 \\ -4x + y &= -12 \end{aligned}$$

although a system may have thousands of equations with thousands of variables. A *solution* to a system of equations is a set of values for the

variables which make all of the equations true. Two methods of solving such a system are the *Addition method* and the *Substitution method*.

– T –

Terms

When the final operation in an expression is multiplication, the expression consists of one term; e.g., ab^2c^3 consists of one term. When the final operation is addition or subtraction, the expression consists of two or more terms; for example, $(a + b)^3 + cd - y$ consists of three terms.

Triangle

A geometric shape consisting of three sides. Its *perimeter* is the sum of the lengths of its three sides, and the sum of the three interior (inside) angles is always 180° .

Trinomial

A *polynomial* with three terms. For example, $3x^5 + 7x^3 + 9$ is a trinomial.

Triple

To multiply a number by 3. The result of tripling 10 is 30, and the result of tripling n is $3n$.

Two Equations in Two Unknowns

A *system of equations* that contains two equations and two variables.

– U –

Undefined

Usually refers to the result of trying to divide by *zero*. Whereas $\frac{0}{7} = 0$, we say that $\frac{6}{0}$ and $\frac{0}{0}$ are undefined. Also, since the slope of a vertical line would

result in a fraction whose denominator is 0, we may also declare that the slope of a vertical line is undefined.

— V —

Variable

A letter of some alphabet whose numerical value can vary from one problem to another. The variable n in the equation $3n = 12$ has a different value than the n in the equation $n + 7 = 100$. See *Constant*.

Vertical Line

A line extending up and down (or north to south). Its equation is always of the form $x = \text{some number}$, and its slope is undefined. Example: Graphing $x = -2$ in the plane results in a vertical line.

— W —

Whole Number

A number in the set $\{0, 1, 2, 3, 4, \dots\}$. It's the set of *natural numbers* with 0 thrown in.

— X —

X-axis

The horizontal axis in a 2-dimensional coordinate system.

X-coordinate

The first of two or more numbers enclosed in parentheses which represent a point in a Cartesian coordinate system. For example, the point $(7, 3)$ in 2-dimensional space has an x -coordinate of 7, as does the point $(7, 0, -1, \pi)$ in 4-dimensional space (the 4th coordinate is the time dimension).

X-intercept

If a graph passes through a point on the x -axis, that point is called an x -intercept.

— Y —

Y-axis

The vertical axis in a 2-dimensional coordinate system.

Y-coordinate

The second of two or more numbers enclosed in parentheses which represent a point in a Cartesian coordinate system. For example, the point (9, 2) in 2-dimensional space has a y -coordinate of 2, as does the point (7, 2, 1, -3 , π) in 5-dimensional space.

Y-intercept

If a graph passes through a point on the y -axis, that point is called a y -intercept.

— Z —

Zero

The origin on the number line, separating the positive from the negative numbers. Zero is neither positive nor negative; it's neutral.

Properties of 0:

$N + 0 = N$ for any number N . See *Additive Identity*

$N \cdot 0 = 0$ for any number N .

The *opposite* of 0 is 0.

0 has no *reciprocal*. [If it did, it would be $\frac{1}{0}$, which is *undefined*.]

If $AB = 0$ then either $A = 0$ or $B = 0$.

Division with 0:

Recall the definition of division:

$$\frac{12}{3} = 4 \text{ because } 4 \times 3 = 12.$$

$$\frac{0}{7} = 0, \text{ because } 0 \times 7 = 0.$$

$$\frac{7}{0} = \text{Undefined}, \text{ because NO number times 0 is 7.}$$

$$\frac{0}{0} = \text{Undefined}, \text{ because EVERY number times 0 is 0.}$$

0 as an Exponent:

Any number (except 0) raised to the zero power is 1. Thus, $7^0 = 1$ and $(-\pi)^0 = 1$. Here's one way to see this.

Let's pretend we don't know what 2^0 is. As in chemistry, where we determine the identity of an unknown compound by mixing it with a known compound, we can multiply 2^0 by something like 2^3 and see what happens.

$$\begin{aligned} 2^0 \times 2^3 &= 2^3 && \text{(add the exponents)} \\ \Rightarrow 2^0 \times 8 &= 8 && (2^3 = 2 \cdot 2 \cdot 2 = 8) \\ \Rightarrow \frac{2^0 \times 8}{8} &= \frac{8}{8} && \text{(divide each side of the equation by 8)} \\ \Rightarrow 2^0 &= 1 && \text{(simplify each fraction)} \end{aligned}$$

If you change the base from 2 to any other number (except 0), the same logic holds. In summary, for any number base x (other than 0),

$$x^0 = 1$$

*“Learning
without
thought
is labor
lost.”*

Confucius

