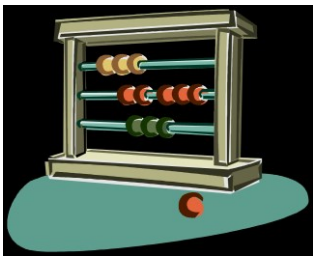
CH 13 – MULTIPLYING AND DIVIDING SIGNED NUMBERS

❑ Introduction



A couple of chapters ago we learned how to solve an equation like $7n = 35$: divide each side of the equation by 7, and conclude that $n = 5$. What if we come across an equation like

$$2x = -14\ ?$$

Our instinct should be to divide each side of the equation by 2 in order to isolate the x. But now we're faced with dividing a negative number (the -14) by a positive number (the 2). Not to worry -- we'll figure it out.

❑ Multiplying Signed Numbers

Recall that the result of multiplying two numbers is called the **product** of the two numbers. So the product of 3 and 5 is 15. Some ways to represent the product of 3 and 5 are

$3 \cdot 5$ $\quad 3 \times 5$ $\quad 3(5)$ $\quad (3)5$ $\quad (3)(5)$ All these products equal 15

Instead of using parentheses to indicate multiplication, square brackets are sometimes used. So [3][5] = 15 also.

Recall the *commutative property of multiplication*, where the order in which you multiply a pair of numbers makes no difference in the product: $ab = ba$ for any numbers a and b.

Also recall the *associative property of multiplication*, where parentheses can be "shifted" without changing the answer:

$$(ab)c = a(bc) \text{ for any numbers } a, b, \text{ and } c$$

1. Positive Times Positive What should 6×4 be? Luckily, the answer is exactly what you would expect. In other words, what we learned as kids still holds: **A positive number times a positive number is positive**.

2. Positive Times Negative This one's not so obvious. Consider the multiplication problem $(7)(-1)$. We'll determine the answer by sneaking up on the problem while it's not looking:

$(7)(2) = 14$	since a positive times a positive is positive
$(7)(1) = 7$	same rule – or – anything times 1 is itself
$(7)(0) = 0$	anything times 0 is 0
$(7)(-1) =$???	what should the product be?

What comes next in the sequence 14, 7, 0, ? Since this sequence of numbers is decreasing by 7 at each step, the next number in the sequence must be –7, and we see that $(7)(-1) = -7$. It appears that **a positive number times a negative number is negative**.

3. Negative Times Positive To calculate $(-3)(4)$, first reverse the order of the factors (commutative property): $(-3)(4) = (4)(-3)$, which is now a product of a positive with a negative. By the previous rule, we know the answer is –12. Thus, **a negative number times a positive number is negative**.

4. Negative Times Negative Now for the most interesting situation, the product of two negative numbers -- for example $(-5)(-1)$. We'll get a running start and see what emerges.

$(-5)(2) = -10$	since a negative times a positive is negative
$(-5)(1) = -5$	same rule – or – anything times 1 is itself
$(-5)(0) = 0$	anything times 0 is 0
$(-5)(-1) =$???	what should the product be?

What comes next in the sequence of answers –10, –5, 0 ? Since this sequence of numbers is <u>increasing</u> by 5 at each step, it follows that the next number is 5, giving us the result:

(–5)(–1) = 5

We've reached the inescapable conclusion that **a negative number times a negative number is <u>positive</u>**!

See if you can deduce the two rules in the following box from the four rules stated above.

Multiplying Signed Numbers:

If the signs are the same, the product is positive.

If the signs are different, the product is negative.

Homework

1. Find the product:

a. (17)(3)	b. (–4)(7)	c. 3(–10)	d. (–3)(–4)
e. –7(–2)	f. 2(–7)	g. –1(8)	h. –1(–9)
i. (1)(–134)	j. (–765)(0)	k. –3 × –4	l. 7 · –10
m. (–18)(–2)	n. 7(–3)	o. –8(7)	p. 3 · –9
q. (–2)(99)	r. (–1)(–7)	s. (7)(–10)	t. (–4)(–5)

2. Find the product:

a. (0.2)(0.3)	b. (–0.1)(0.1)	c. (2.1)(–3)	d. (–0.3)(–0.4)
e. (–0.3)(5)	f. 2(–1.77)	g. (–0.1)(–0.2)	h. 17(–0.2)

3. Find the product in reduced form:

a. $\frac{2}{3}\times\frac{1}{5}$ b. $-\frac{2}{3}\cdot\frac{3}{4}$ c. $\frac{4}{5}\left(-\frac{5}{4}\right)$ d. $(-2)\left(-\frac{7}{2}\right)$

e. $\frac{4}{5}\times-\frac{10}{3}$ f. $\left[-\frac{1}{2}\right]\left[\frac{2}{99}\right]$ g. $\left(-\frac{2}{3}\right)\left(-\frac{3}{2}\right)$ h. $10\left(-\frac{3}{40}\right)$

4. Find the product:

Example: $(-2)(3)(-4)$
$= (-6)(-4)$ (multiply the first two factors)
$= 24$

a. $(-2)(-3)(4)$ b. $7\times-6\times3$ c. $(-1)(-2)(-3)(-4)$

d. $4\cdot4\cdot4(-1)(3)$ e. $5(-1)(-1)(-2)$ f. $4(-3)(2)(-8)$

g. $(-2)(-1)(-3)$ h. $(-7)(6)(2)(-1)$ i. $(-1)(-1)(-1)(8)$

j. $(-1)(-3)(-5)(-7)$ k. $-3(-4)(5)2$ l. $(-2)(-2)(-1)(-3)(-5)0$

❑ Dividing Signed Numbers

The secret to the division rules for signed numbers is the fact that division is checked by multiplication. For example,

$$\frac{56}{7} = 8, \text{ precisely because } 8\times7 = 56$$

1. Positive Divided by Positive

$\frac{6}{2} = 3$, since $(3)(2) = 6$.

Therefore, **a positive number divided by a positive number is positive**.

2. Negative Divided by Positive

$\frac{-6}{2} = -3$, because $(-3)(2) = -6$.

Thus, **a negative number divided by a positive number is negative**.

3. Positive Divided by Negative

$\frac{6}{-2} = -3$, which is checked by seeing that $(-3)(-2) = 6$.

Conclusion: **a positive number divided by a negative number is negative**.

4. Negative Divided by Negative

$\frac{-6}{-2} = 3$, which is confirmed by the fact that $(3)(-2) = -6$.

We see that **a negative number divided by a negative number is positive**.

NOTES: 1. Do these four rules for dividing signed numbers remind you of anything? The rules for dividing signed numbers are the same as the rules for multiplying signed numbers.

2. Note that $\frac{-10}{2} = \frac{10}{-2} = -\frac{10}{2}$,

because each of these three division problems has a quotient of –5. In general, the following three fractions are equal:

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$$

Homework

5. Using the logic above, which explains how signed numbers are divided, explain why each division problem is correct:

a. $\frac{22}{11} = 2$

b. $\frac{-21}{7} = -3$

c. $\frac{88}{-22} = -4$

d. $\frac{-100}{-20} = 5$

Here's the bottom line for dividing signed numbers:

Dividing Signed Numbers:

If the signs are the same, the quotient is positive.

If the signs are different, the quotient is negative.

EXAMPLE 1: **Find each quotient:**

A. $\frac{45}{-9} = -\mathbf{5}$ (positive divided by negative is negative)

B. $\frac{-250}{-25} = \mathbf{10}$ (negative divided by negative is positive)

C. $\frac{-21}{15} = -\frac{21}{15} = -\frac{\mathbf{7}}{\mathbf{5}}$ (negative divided by positive is negative)

D. $\frac{-8}{-12} = \frac{8}{12} = \frac{\mathbf{2}}{\mathbf{3}}$ (negative divided by negative is positive)

E. $\frac{18}{-7} = -\frac{\mathbf{18}}{\mathbf{7}}$ (positive divided by negative is negative)

F. $\frac{-4}{-9} = \frac{\mathbf{4}}{\mathbf{9}}$ (negative divided by negative is positive)

Homework

6. Find the quotient (leave answers as simplified fractions):

a. $\frac{30}{5}$ b. $\frac{60}{-10}$ c. $\frac{-100}{20}$ d. $\frac{-14}{-2}$

e. $\frac{3}{6}$ f. $\frac{-18}{4}$ g. $\frac{24}{-5}$ h. $\frac{-34}{-24}$

i. $\frac{-1}{9}$ j. $\frac{-345}{-345}$ k. $\frac{10}{-10}$ l. $\frac{-120}{-70}$

7. Evaluate each expression:

a. $\frac{5-1}{1-2}$ b. $\frac{10(-2)}{-4}$ c. $\frac{3^2-9}{7}$

d. $\frac{18-2}{2-18}$ e. $\frac{1+2+3}{-2-4}$ f. $\frac{3^2-1^3}{6-5}$

8. Evaluate each expression:

Example: $4 - 5[-3 + 2(3 - 6)]$

$= 4 - 5[-3 + 2(-3)]$ (start with innermost parentheses)

$= 4 - 5[-3 - 6]$ (multiply before adding)

$= 4 - 5[-9]$ (now for the brackets)

$= 4 + 45$ (multiply before subtracting)

$= \mathbf{49}$ (and we're done)

Evaluate each expression:

a. $2[5 - 9(-2)]$ b. $-3[1 + 2(-5)]$

c. $4 + 3[2 - 3(4)]$ d. $4 - 5[1 + 5(1 - 4)]$

e. $10 - 8[2(3) - 4(2)]$ f. $7 - 2[8 - 2(3 - 4)]$

g. $4 - 7[-5 - (2 - 9)]$ h. $10 - 2[10 - 2(3 - 7)]$

Review Problems

9. A negative divided by a negative is positive. Explain why.

10. a. $(-5)(-2)(-3) =$ b. $\frac{45}{-9} =$ c. $\frac{-80}{25} =$

d. $\frac{-12}{-48} =$ e. $\frac{2-8-10}{12-1-5} =$

11. a. $(-5)(2)(-4) =$ b. $\frac{-45}{-9} =$ c. $\frac{80}{-15} =$

d. $\frac{-36}{-48} =$ e. $\frac{2-8+10}{12+1-5} =$

12. $4 - 3[2(-3) - 5(8 - 6)] =$

The entire chapter can be summarized like this:
For both multiplying and dividing,

Same signs $\Rightarrow$ positive answer

Different signs $\Rightarrow$ negative answer

Solutions

1. a. 51 b. –28 c. –30 d. 12 e. 14 f. –14
g. –8 h. 9 i. –134 j. 0 k. 12 l. –70
m. 36 n. –21 o. –56 p. –27 q. –198 r. 7
s. –70 t. 20

2. a. 0.06 b. –0.01 c. –6.3 d. 0.12 e. –1.5 f. –3.54
g. 0.02 h. –3.4

3. a. $\frac{2}{15}$ b. $-\frac{1}{2}$ c. -1 d. 7 e. $-\frac{8}{3}$ f. $-\frac{1}{99}$
g. 1 h. $-\frac{3}{4}$

4. a. 24 b. −126 c. 24 d. −192 e. −10 f. 192
g. −6 h. 84 i. −8 j. 105 k. 120 l. 0

5. a. $(2)(11) = 22$ b. $(-3)(7) = -21$
c. $(-4)(-22) = 88$ d. $(5)(-20) = -100$

6. a. 6 b. −6 c. −5 d. 7 e. $\frac{1}{2}$
f. $-\frac{9}{2}$ g. $-\frac{24}{5}$ h. $\frac{17}{12}$ i. $-\frac{1}{9}$ j. 1
k. −1 l. $\frac{12}{7}$

7. a. −4 b. 5 c. 0 d. −1 e. −1 f. 8

8. a. 46 b. 27 c. −26 d. 74
e. 26 f. −13 g. −10 h. −26

9. $\frac{\text{neg}}{\text{neg}}$ = pos because when the division is checked by multiplication, we confirm that pos × neg = neg ✓

10. a. −30 b. −5 c. $-\frac{16}{5}$ d. $\frac{1}{4}$ e. $-\frac{8}{3}$

11. a. 40 b. 5 c. $-\frac{16}{3}$ d. $\frac{3}{4}$ e. $\frac{1}{2}$

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"Let us think of education as the means of developing our greatest abilities, because in each of us there is a private hope and dream which, fulfilled, can be translated into benefit for everyone and greater strength of the nation."

– John F. Kennedy