# Table of Contents



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|---|---|---|
| **ABSOLUTE VALUE** | ***The Basics*** | The absolute value of a quantity is never _____.<br>$\lvert 7-9 \rvert =$ |
| | ***Equations*** | Solve for $n$: $\lvert 7n-20 \rvert = 45$ |
| | ***Graphing*** | Graph: $y = \lvert 2x-3 \rvert + 4$ |
| **APPLICATIONS** | ***Investing in the Stock Market*** | One part of a \$41,500 investment is designed to be four times the other part of the investment. How will the money be allocated? That is, find the two parts of the investment. |

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| | ***Number Word Problems*** | I'm thinking of a number. If 7 is subtracted from the number, and then that result is multiplied by 10, the final result is 150. What is the number?<br><br>Find a number given that 3 times the difference of the number and 1 is 13 more than the number. |
| **B** | **B** | **B** |
| **BINOMIAL THEOREM** | ***The Binomial Theorem*** | How many terms are there in the expansion of $(a + b)^{56}$?<br><br>Expand: $(a + b)^9$ |
| **C** | **C** | **C** |
| **CIRCLES** | ***Pre-Algebra Level*** | The definition of $\pi$: $\frac{C}{d}$<br><br>The radius of a circle is 10. Find the circumference and the area. |
| | ***Algebra Level*** | Derive the formula $C = 2\pi r$.<br><br>The area of a circle is $100\pi$. What is its circumference? |
| | ***Center at the Origin*** | Find the center and radius of the circle $x^2 + y^2 = 12$.<br><br>Find the equation of the circle with center at the origin and with a radius of $4\sqrt{7}$. |

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| | ***Center off the Origin*** | Find the center and radius of $(x+3)^2 + (y-7)^2 = 9$.<br><br>Determine the equation of the circle with center at (2, –11) and radius $5\sqrt{2}$.<br><br>Calculate the center and radius of the circle<br>$x^2 + y^2 - 8x + 6y = 3$ |
| | ***Advanced*** | Find the equation of the circle whose center is (9, –12) and whose diameter is $8\sqrt{5}$.<br><br>Find the equation of the circle given that its center is at the point (10, –4) and that it passes through the point (8, 3).<br><br>Find the equation of the circle given that the endpoints of a diameter are the points (–5, –2) and (3, 4).<br><br>Find the equation of the top-half of the circle $x^2 + y^2 = 10$. |
| **COMBINATIONS** | ***Combinations*** | How many ways are there to choose 5 numbers in a lottery containing the numbers 1 through 100? |
| **COMPLETING THE SQUARE** | ***Preparing*** | Solve for $x$: $(x+14)^2 = 100$ See ***Solving Quadratics by Taking Square Roots***<br><br>Factor: $y^2 - \frac{6}{7}y + \frac{9}{49}$<br><br>Find the *magic number*: $n^2 + 15n$ |
| | ***Solving Quadratics*** | Solve by completing the square: $3x^2 - 5x + 1 = 0$ |

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| **CONIC SECTIONS** | ***The Circle*** | Use the definition of the circle with center $(h, k)$ and radius $r$ to create the formula $(x-h)^2 + (y-k)^2 = r^2$. |
| | ***The Parabola*** | This chapter is based on the definition: *A parabola is the set of points in the plane which are equidistant from a given point and a given line.* $y = \frac{1}{4p}x^2$.<br>Find the vertex, focus, and directrix of the parabola $y = 2x^2 - 8x + 5$. |
| | ***The Ellipse*** | Find the center, the domain, and the range of the ellipse, and then graph it:<br>$$25x^2 + 9y^2 - 50x - 108y + 124 = 0$$ |
| **CONSECUTIVE INTEGERS** | ***Linear*** | Find four consecutive odd integers such that the largest is 3 less than twice the smallest. |
| | ***Quadratic*** | Find four consecutive negative even integers such that the product of the smallest and the largest is 40. |
| **CONSTANTS** | ***Finding the Numbers*** | Find two numbers with a product of 16 and a difference of 6.<br>If I quadruple the number, and then subtract 8, the result is 40. |
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| **D** | **D** | **D** |
| **DISTANCE, RATE, AND TIME** | **Go to *Motion Problems*** | |

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| **DISTANCE ON THE LINE AND IN THE PLANE** | ***Distance on the Line and in the Plane*** | What is the formula for finding the distance between the points $a$ and $b$ on the number line?<br>Use a triangle and the Pythagorean Theorem to find the distance between two points in the plane.<br>Use the Distance Formula to find the distance between two points in the plane. |
| **DISTRIBUTIVE PROPERTY** | ***The Basics*** | For any numbers, $a(b + c) = ab + ac$. Simplify each expression:<br>A. $3(4x + 5y - 7)$ B. $7(A \cdot 4)$ C. $2(x - 3) + 7x$ |
| | ***Multiplying Binomials*** | Simplify each expression:<br>A. $(2x + 5)(x - 3)$ B. $(7n + 1)(7n - 1)$<br>C. $(4w - 3)^2$ D. Prove that $(x + y)^2 \neq x^2 + y^2$ |
| | ***Solving Equations*** | Solve for $w$: $8(5w - 1) - 6(-w - 5) = 5(8 - 4w) + 6(9 + 8w)$ |
| **DOMAIN** | ***Domain*** | Find the *domain* of each function:<br>A. $y = 3x^2 + 7$ B. $y = \sqrt{2 - 6x}$<br>C. $y = 7$ D. $y = \dfrac{2x - 4}{x^2 + 5x + 6}$ |
| **E** | **E** | **E** |
| ***e*** | ***The Number e*** | New limit notation<br>banking<br>Graphing |

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| **ELLIPSES** | ***The Ellipse (Center at the Origin)*** | This chapter does not involve the *constant sum of the distances to 2 points* definition of the ellipse.<br>Graph the following ellipse by plotting the four intercepts and four additional points:<br>$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$ |
| **EQUATIONS** | ***Introduction*** | |
| | ***Distance and the Origin*** | |
| | ***Linear with Distributing*** | |
| | ***Absolute Value*** | |
| | ***Quadratics by Factoring*** | |
| | ***Quadratics by Taking Square Roots*** | |
| | Quadratics by Completing the Square | |

| | | |
|---|---|---|
| | The Quadratic Formula, Rational Solutions | |
| | The Quadratic Formula, Irrational Solutions | |
| | Square Roots | |
| | Cubics and Quartics | |
| | ***Fractional*** | |
| | ***Exponential*** | Solve for $x$: $9^{4x-1} = 27$<br>Solve for $a$: $125^{-10a} = \frac{1}{25}$ |
| **EXPONENTIAL EQUATIONS** | ***Exponential Equations*** | |
| **EXPONENTIAL FUNCTIONS** | ***Exponential Functions*** | |
| **EXPONENTS** | ***Introduction*** | |
| | ***More*** | |

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| | *The Five Laws* | |
| | *Negative* | |
| | *Fractional* | |
| **F** | **F** | **F** |
| **FACTORIALS** | *Factorials* | SPLIT INTO THE COUNTING PRINCIPLE AND FACTORIALS?<br><br>Anita has 3 skirts, 4 blouses, and 2 caps. How many different outfits can Anita create from these items?<br><br>You have to line up 12 kindergartners at the door in all possible ways. It takes one second for each possible lineup. How many years will it take you to accomplish this feat? |
| **FACTORING** | *The GCF (Greatest Common Factor)* | |
| | *Quadratics – An Intro* | |
| | *Quadratics – The Real Deal* | |
| | *Complete Factoring* | Reducing Fractions and Solving Quadratics |

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| | ***Advanced Factoring*** | Factor each expression completely:<br>$9a^4 - 37a^2 + 4$ $\quad$ $x^2(u - w) - 100(u - w)$<br>$x^3 - 7x^2 - 9x + 63$ $\quad$ $x^2 + 6x + 9 - y^2$ |
| | ***Sum and Difference of Cubes*** | Long division<br>Formula<br>Find the cube roots (real and complex) of 8. |
| **FORMULAS** | ***Using*** | |
| | ***Solving*** | Solve for $x$: $\frac{bx+c}{c} - L = M$ |
| | ***Creating*** | from English statements |
| **FRACTIONAL EQUATIONS** | ***Solving*** | Example of Solving<br>See Motion (last part) for applications |
| **FRACTIONS** | ***Introduction*** | |
| | ***More*** | Signed fractions, Powers, Roots, and Equations |
| | ***Reducing with the GCF*** | |
| | ***Reducing Quadratics*** | Reduce: $\frac{m^2+10m+25}{m^2-25}$ |

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| | *Adding and Subtracting, Part I* | |
| | *Adding and Subtracting, Part II* | |
| | *Multiplying and Dividing* | |
| **Fractions and Decimals** | *Fractions and Decimals* | |
| **Functions** | *Tables and Mappings* | |
| | *Formulas and Graphs* | |
| | *Notation and Composition* | |
| | *Piecewise* | |
| | *Rational* | |
| G | G | G |

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| **GCF** | ***GCF - Equations and Formulas*** | |
| **GEOMETRY** | ***An Introduction*** | |
| | ***Squares and Circles*** | If the perimeter of a square is 44, what is its area?<br>If the area of a circle is $169\pi$, what is its circumference? |
| | ***Triangles and Perimeter*** | The third side of a triangle is 3 less than the first side, while the first side is 12 less than 2 times the second side. If the perimeter is 58, what is the length of each side? |
| | ***Triangles and Angles*** | The second angle of a triangle is 38° less than 4 times the first angle. The third angle is 13° more than the second angle. What is the measure of each angle? |
| **GRAPHING** | ***The Coordinate Plane*** | This is also called the **Cartesian Plane**. |
| | ***From Graph to Equation*** | |

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| **GROWTH AND DECAY** | ***The Growth and Decay Formula*** | Uses the formula: $A = A_0 e^{kt}$<br><br>Assuming an initial population of 3,902, and a *growth* rate of 14% per year, determine the population in 10 years.<br><br>Starting with 215 grams of plutonium, and assuming an annual *decay* rate of 15%, calculate the number of grams remaining after 26 years. |
| **H** | **H** | **H** |
| **HYPERBOLAS** | ***The Hyperbola (Center at the Origin)*** | |
| **I** | **I** | **I** |
| **INEQUALITIES** | ***Inequalities*** | Express as an inequality: Your height, *h*, can be no taller than 48 inches.<br><br>Solve for *y*: $\frac{9-y}{-2} \geq -2$<br><br>Express as a double inequality: The score, *s*, must be between 100 pts and 300 pts, excluding the 100 and the 300. |
| | ***Graphing in the Plane*** | |
| | ***Absolute Value and Quadratic*** | Solve for *x*: $\|3x+7\| > 10$<br><br>Solve for *a*: $a^2 + 3a - 10 \leq 0$ |

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| **INTERCEPTS** | ***Go to Lines or Parabolas*** | move this to a separate section of links, along with things like *horizontal line* and *binomial*. |
| **INTERVAL NOTATION** | ***Interval Notation*** | |
| **J** | **J** | **J** |
| **K** | **K** | **K** |
| **L** | **L** | **L** |
| **LIKE TERMS** | ***Combining*** | Simplify: $3x + 7y - 9 - 2y - 13x + 14$ |
| | ***Solving Equations*** | Solve for $t$: $7t + 9 - 2t - 1 = 17t - 18t + 22$<br>Here are two cell phone plans:<br>Plan #1 Signup fee: \$19.00 Per minute: \$0.65<br>Plan #2 Signup fee: \$6.00 Per minute: \$0.90<br>How many minutes of talk time would result in the same charge for each plan? |
| **LINES** | ***Graphing*** | |
| | ***Intercepts*** | |

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| | ***Slope - Rise Over Run*** | |
| | ***Slope - $\Delta y / \Delta x$*** | |
| | ***Creating slope/int*** | |
| | ***Finding $y = mx+b$*** | includes rational slopes and $y$-intercepts |
| | ***Point/Slope*** | $y - y_1 = m(x - x_1)$ |
| | ***Parallel and Perpendicular Lines*** | |
| | ***Graphing Revisited*** | |
| | ***Special Lines*** | Graph the line $x = 7$.<br>What is the slope of the line $y = -9$?<br>Give the equation of the line which is perpendicular to the line $y = \pi$? |

<table>
<tr><td rowspan="1">Logic</td><td>Logic</td><td>Implications<br>Negation, Converse, and Contrapositive<br>AND & OR<br>Definitions, Axioms, and Theorems<br>Examples and Counterexamples</td></tr>
<tr><td rowspan="4">Logarithms</td><td>Calculating Logarithms</td><td>Translate $\log_6 36 = 2$ to exponent form.<br>Calculate: $\log_2\left(\frac{1}{8}\right)$<br>Find the pH of a substance whose hydrogen ion concentration is $3.8 \times 10^{-9}$.</td></tr>
<tr><td>Log Functions</td><td>Find the domain: $y = \log(x^2 - 5x + 6)$<br>Graph and specify the domain, intercepts, and relevant limits:<br>$g(x) = \ln(2x - 6)$<br>The sound produced by a plane engine has an intensity of $7.1 \times 10^3$ W/m$^2$. Find the decibel value of this sound.</td></tr>
<tr><td>The Laws of Logs</td><td></td></tr>
<tr><td>Log Equations</td><td></td></tr>
<tr><td>Long Division</td><td>Dividing Polynomials</td><td>Divide: $\frac{10x^2 + 5x + 2}{5x}$<br>Divide: $\frac{14x^2 + 3x - 7}{2x - 1}$</td></tr>
</table>

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| **Midpoint** | ***Midpoint*** | |
| **Mixture Problems** | ***Producing Widgets*** | The unit cost of red widgets is \$6 and the unit cost of white widgets is \$8. If the goal is to make 25 widgets for a total of \$170, how many widgets of each color should we make? |
| | ***Coins*** | Lisa has 7 times as many dimes as nickels. If the total value of the coins is \$3.75, how many of each coin does Lisa have? |
| | ***Nuts*** | Trader Joe's has 4 lbs of walnuts which are priced at \$24/lb. How many pounds of almonds which are priced at \$10/lb should be mixed with the walnuts to get a mixture that will sell for \$18/lb?<br><br>Peanut Palace has 28 lbs of walnuts which are priced at \$19/lb, and mixes them with 12 lbs of almonds which are priced at \$29/lb. Find the *unit price* of the mixture. |
| | ***Percent, Part I*** | A chemist mixes 8 mL of a 37% glycerin solution with some 57% glycerin solution. If the final mixture is 49% glycerin, how many mL of the 57% glycerin solution were used?<br><br>A 63% acid solution is created by mixing 3 liters of an 87% acid solution with 12 liters of another acid solution. Find the concentration of the other acid solution. |
| | ***Percent, Part II*** | A detective wants to mix some pure poison with some 20% poison solution. How many pounds of each substance must she use to get a 25-pound mixture that is 52% poison? |

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| **MODELING** | ***Temperature Formulas*** | |
| | ***Linear Using Given Information*** | |
| | ***Using the Equation of a Line*** | |
| **MOTION PROBLEMS** | ***Introduction*** | Moe traveled at a rate of 120 km/hr for 12 hours. Find Moe's distance. |
| | ***Opposite Directions*** | Two joggers leave the same place and jog in opposite directions. The speed of one of the joggers is 9 mph more than 5 times the other. In 7 hours they are 357 miles apart. Find the speed of each jogger. |
| | ***Pursuit*** | A rowboat leaves the harbor traveling 26 mph. Seven hours later a speedboat begins to pursue the rowboat at a speed of 39 mph. How many hours after the speedboat leaves the harbor will it catch up with the rowboat? |
| | ***Round Trip*** | A helicopter traveled the hospital to the battlefield at a speed of 36 mph and returned at a speed of 24 mph. If the entire trip took 20 hours, find the travel times to and the battlefield. |

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| | ***Same Direction*** | Lucy and Ethyl leave the mall at the same time and head in the same direction. Ethyl's speed is 4 mph more than 4 times Lucy's speed. Four hours later Ethyl is 604 miles ahead of Lucy. Find the speeds of Lucy and Ethyl. |
| | ***Two-Part Journey*** | A 556-mi trip took a total of 14 hours. The speed was 38 mph for the first part of the trip, and then increased to 42 mph for the rest of the trip. How many hours were traveled at each speed? |
| | ***Problems Using Fractional Equations*** | Phil hikes up a 32-mile mountain trail. He then rents a pair of skis and skis back down the trail. His skiing speed is 8 mph faster than his hiking speed. If Phil spent a total of 6 hours on the trail, what were his hiking speed and his skiing speed?<br><br>Raya can go 9 miles upstream in her motorboat in the same time she can go 33 miles downstream. If the speed of her boat is 7 mph in still water, what is the rate of the current? |
| **N** | **N** | **N** |
| **O** | **O** | **O** |
| **OPTIMIZATION** | ***Optimization*** | A hard drive company found that the profit they make on their hard drive is given by the formula<br><br>$P = -4h^2 + 200h + 1000$<br><br>where $P$ is profit and $h$ is the number of hard drives sold. Find the number of hard drives that the company needs to produce in order to **maximize** their profit. Also determine the maximum profit. |

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| **ORDER OF OPERATIONS** | *Order of Operations* | |
| **P** | **P** | **P** |
| **PARABOLAS** | *Graphing* | Graph the parabola $y = x^2 + 2x - 8$, and then use your graph to determine all the intercepts and the vertex. |
| | *Intercepts, Rational* | Find all intercepts of the parabola $y = 15x^2 - 34x + 15$. |
| | *Intercepts, Irrational* | Find all intercepts the parabola $y = -3x^2 + 5x - 1$.<br>Prove that the parabola $y = x^2 + x + 2$ has NO $x$-intercepts. |
| | *Vertex* | |
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| **POLYNOMIALS** | *Polynomials* | |
| **PROPORTIONS** | | |
| **PYTHAGOREAN THEOREM** | *Finding the Hypotenuse* | |
| | *Finding a Leg* | |
| | *With Radicals* | |
| **Q** | **Q** | **Q** |

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| **Quadratic Equations** | ***By Factoring*** | Also See: *Quadratic Formula*<br>*Completing the Square* |
| | ***Taking Square Roots*** | Solve for $y$ by taking square roots: $(5y-3)^2 = 50$ |
| **Quadratic Formula** | ***Preparing*** | Verify that $-5$ and $2$ are solutions of $x^2 + 3x - 10 = 0$.<br>Put $2x^2 = 4x + 3$ into standard form and determine the values of the coefficients $a$, $b$, and $c$.<br>Evaluate the expression $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$<br>for the values $a = 1$, $b = -5$, $c = 6$ |
| | ***Deriving*** | Begin with $ax^2 + bx + c = 0$, complete the square, and end up with $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. |
| | ***Rational Solutions*** | |
| | ***Irrational Solutions*** | |
| | ***Applications*** | |
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| **R** | **R** | **R** |

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| **RADICALS** | *Square Roots* | |
| | *Higher Roots* | |
| | *More Operations* | |
| | *Advanced* | Simplify: $\sqrt{288a^{46}b^{19}}$<br>Simplify: $\sqrt[4]{\sqrt[3]{\sqrt{2}}}$ |
| **RATIOS** | | |
| **REAL NUMBERS** | *The Real Numbers* | |
| **S** | **S** | **S** |
| **SCIENTIFIC NOTATION** | *Scientific Notation* | |
| **SEQUENCES** | *Sequences* | |
| **SERIES** | *Series* | Evaluate: $\sum_{k=2}^{5}(2k+1)$<br>Convert to sigma notation |

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| **Sets** | ***Sets*** | Elements and Subsets, the Null Set<br>Finding the number of elements in a set (its *cardinality*)<br>Union and Intersection<br>Finding all subsets |
| **Signed Numbers** | ***Introduction*** | |
| | ***Adding and Subtracting*** | |
| | ***Multiplying and Dividing*** | |
| | ***Division with Zeros and Equations*** | |
| | ***More on Equations*** | |
| **Systems of Equations** | ***Introduction*** | |
| | ***Substitution*** | |
| | ***Elimination*** | |
| | ***Graphing*** | |
| | ***Three Variables*** | |

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| T | T | T |
| U | U | U |
| UNIT CONVERSIONS | *Unit Conversions* | |
| V | V | V |
| VARIABLES | *Variables and Identities* | Use an *example* to help verify that $(x+5)(x-5) = x^2 - 25$ *is* an identity.<br>Use a *counterexample* to prove that $(x+3)(x+2) = x^2 + 6$ is *not* an identity. |
| VARIATION | *Direct and Inverse Variation* | Write as a formula: $y$ varies directly as the product of $x$ and $z$, and inversely as the fourth power of $w$.<br>The volume of a gas varies directly as its temperature, and inversely as its pressure. If the volume is 14 when the temperature is 9 and the pressure is 12, find the volume when the temperature is 5 and the pressure is 16. |
| VELOCITY | *Average Velocity* | Let $s$ be the position function given by $s(t) = t^3$. Find the *average velocity* during the time interval from $t = 3$ to $t = 10$. |
| W | W | W |
| X | X | X |

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| Y | Y | Y |
| Z | Z | Z |
| ZERO | *The Number Zero* | |

G L O S S A R Y

**Additive Identity:** A fancy term for the number **0**: $n + 0 = n$ for any number $n$.

Applications: [Mixture Problems] [Motion Problems]

**Average Velocity:** Change in position divided by change in time, $\bar{v} = \dfrac{\Delta s}{\Delta t}$.

**Binomial:** A polynomial with two terms. Examples: $7x^2 + 9$ and $-a^2b^3 + 9ab^4$

**Circle Equation:** The circle with center $(h, k)$ and radius $r$ has the formula $(x-h)^2 + (y-k)^2 = r^2$.

**Circumference:** The distance around a circle (the circle's perimeter), $C = 2\pi r$.

**Coefficient:** It's "how many" of something you have. For example, the coefficient of the monomial $7x^2y^3$ is **7**.

**Combination Formula:** "*n choose k*" $= \dbinom{n}{k} = \dfrac{n!}{k!(n-k)!}$ [*Combinations*]

**Counting Principle:** [*Factorials*]

**Diameter:** The distance from one point on a circle to another, through the center, $d = 2r$.

**Directly Proportional** [*Direct and Inverse Variation*]

**Distance Formula – on the Line:** The distance between $a$ and $b$ is $|a-b|$.

**Distance Formula – in the Plane:** The distance between $(x_1, y_1)$ and $(x_2, y_2)$ is $\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$.

**Division by Zero:** $\dfrac{7}{0}$ and $\dfrac{0}{0}$ are undefined, but $\dfrac{0}{12} = 0$.

**Domain:** The set of all possible inputs to a function. [*Domain*]

**Equilateral Triangle:** A triangle with three equal sides. It's also *equiangular* – all angles are 60°.

**Isosceles Triangle:** A triangle with at least two equal sides. Notes: An equilateral triangle is isosceles and at least two of the angles must be equal.

**Inversely Proportional** [*Direct and Inverse Variation*]

**Like Terms:** $7n$ and $2n$ are like terms. $5x^2$ and $-7x^2$ are like terms. $3w$ and $4w^2$ are NOT like terms.

**Magic Number:** The quantity that is added to each side of a quadratic equation to "complete the square."

The magic number for $ax^2 + bx + c = 0$ is $\frac{b^2}{4a^2}$.

Opposite:

Quadratic Equation, Standard Form: $ax^2 + bx + c = 0$ (where $a \neq 0$)

Radius: The distance from the center of a circle to any point on the circle, $r = \frac{d}{2}$, where $d$ = diameter.

Reciprocal:

Scalene Triangle: A triangle with NO equal sides.

Simultaneous Equations: Another term for *Systems of Equations*.

Trinomial: A polynomial with three terms. Typical example: $-4x^5 + 7x^2 - 9$

Zeros of a Function: If $f$ is a function, then $\boldsymbol{x}$ is a **zero** of the function if $f(\boldsymbol{x}) = 0$. The zeros of a function are merely the $x$-intercepts of the graph of the function.